# Further development of several equations concerning the Ramanujan Manuscript Book 1. New possible mathematical connections with various sectors of Number Theory, Dark Energy, Dark Matter and Supersymmetry Breaking. VII

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## Abstract

*In this research thesis, (part VII) we analyze further equations concerning the Ramanujan Manuscript Book 1. We obtain new possible mathematical connections with various sectors of Number Theory, Dark Energy, Dark Matter and Supersymmetry Breaking*

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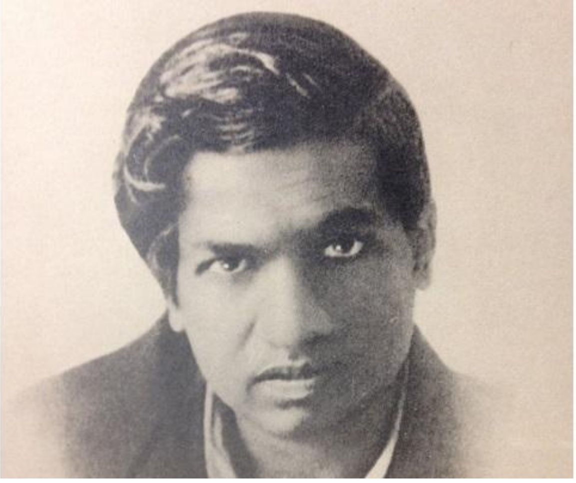
**Srinivasa Ramanujan (1887-1920)**



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

**Vesuvius landscape with gorse – Naples**

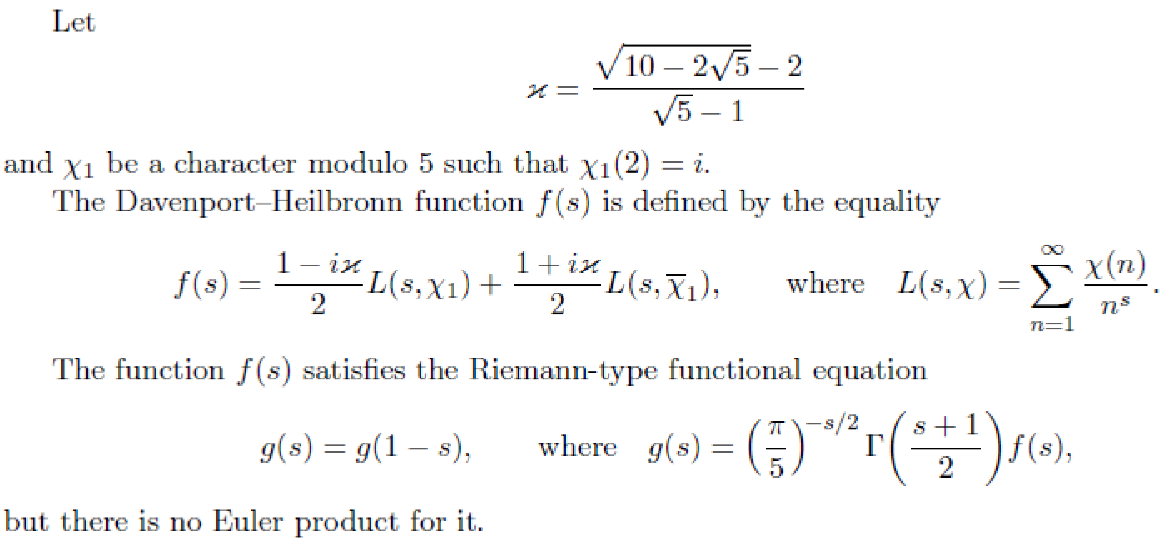
https://www.pinterest.it/pin/95068242114589901/

From:

**On the Zeros of the Davenport Heilbronn Function**
*S. A. Gritsenko* - Received May 15, 2016 - ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2017, Vol. 296, pp. 65–87.

**A. A. Karatsuba**, "On the zeros of arithmetic Dirichlet series without Euler product," Izv. Ross. Akad. Nauk, Ser. Mat. 57 (5), 3–14 (1993)

We have:



Let

$$\varkappa = \frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

and $\chi_1$ be a character modulo 5 such that $\chi_1(2) = i$.

The Davenport–Heilbronn function $f(s)$ is defined by the equality

$$f(s) = \frac{1-i\varkappa}{2}L(s,\chi_1) + \frac{1+i\varkappa}{2}L(s,\overline{\chi}_1), \qquad \text{where} \quad L(s,\chi) = \sum_{n=1}^{\infty}\frac{\chi(n)}{n^s}.$$

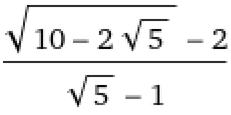
The function $f(s)$ satisfies the Riemann-type functional equation

$$g(s) = g(1-s), \qquad \text{where} \quad g(s) = \left(\frac{\pi}{5}\right)^{-s/2}\Gamma\left(\frac{s+1}{2}\right)f(s),$$

but there is no Euler product for it.

$$(\sqrt{10-2\sqrt{5}}-2)/(\sqrt{5}-1) = \kappa$$

**Input:**



$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

## Decimal approximation:

0.2840790438404122960282918323931261690910880884457375827591626661

...

0.28407904384…. = κ

## Alternate forms:

$$\frac{1}{4}\left(\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2\right)$$

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$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(\sqrt{10-2\sqrt{5}}-2\right)$$

---

$$\frac{1}{2}\left(-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

## Minimal polynomial:

$$x^4+2x^3-6x^2-2x+1$$

## Expanded forms:

$$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}-\frac{2}{\sqrt{5}-1}$$

---

$$\frac{1}{4}\sqrt{10-2\sqrt{5}}+\frac{1}{4}\sqrt{5\left(10-2\sqrt{5}\right)}+\frac{1}{2}\left(-1-\sqrt{5}\right)$$

For ((((√(10-2√5) -2))⁄((√5-1)))) = 8πG; G = 0.011303146014

Indeed:

((((√(10-2√5) -2))⁄((√5-1))))/(8π)

**Input:**

$$\frac{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}{8\pi}$$

**Result:**

$$\frac{\sqrt{10-2\sqrt{5}}-2}{8\left(\sqrt{5}-1\right)\pi}$$

**Decimal approximation:**

0.0113031460140052147973750129442035744685760313920017808594909667…

0.01130314…. = g (gravitational coupling constant)

**Property:**

$\frac{-2+\sqrt{10-2\sqrt{5}}}{8\left(-1+\sqrt{5}\right)\pi}$ is a transcendental number

**Alternate forms:**

$$\frac{\sqrt{10-2\sqrt{5}} - 2\sqrt{5} + \sqrt{5\left(10-2\sqrt{5}\right)} - 2}{32\pi}$$

$$-\frac{1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

$$\frac{-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

## Expanded forms:

$$-\frac{1}{16\pi} - \frac{\sqrt{5}}{16\pi} + \frac{\sqrt{10-2\sqrt{5}}}{32\pi} + \frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{32\pi}$$

$$\frac{\sqrt{10-2\sqrt{5}}}{8\left(\sqrt{5}-1\right)\pi} - \frac{1}{4\left(\sqrt{5}-1\right)\pi}$$

## Series representations:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)} = \frac{-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}}{8\pi\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(5-z_0\right)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We note that:

(((√(10-2√5) -2))/((√5-1)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2)))

**Input:**

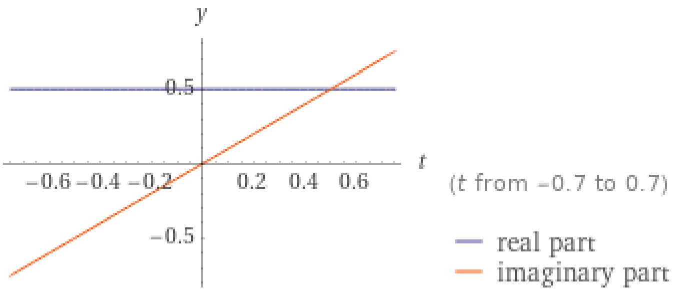
$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\times\frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

$i$ is the imaginary unit

**Exact result:**

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

**Plot:**



## Alternate form assuming t>0:

$$\frac{i\sqrt{10-2\sqrt{5}}\;t}{\sqrt{2\left(5-\sqrt{5}\right)}-2}-\frac{2\,i\,t}{\sqrt{2\left(5-\sqrt{5}\right)}-2}+$$
$$\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-\frac{\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-$$
$$\frac{\sqrt{5}}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}+\frac{1}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

## Alternate forms:

$$\frac{1}{8}\left(1+\sqrt{5}\right)\left(2\,i\sqrt{2\left(3-\sqrt{5}\right)}\;t+\sqrt{5}-1\right)$$

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$$\frac{1}{2}\left(1+2\,i\,t\right)$$

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$$\frac{1}{2}+i\,t$$

*1/2+it = real part of every nontrivial zero of the Riemann zeta function*

**Derivative:**

$$\frac{d}{dt}\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\right)=i$$

**Indefinite integral:**

$$\int\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\,dt=\frac{t}{2}+\frac{i\,t^2}{2}+\text{constant}$$

And again:

(((√(10-2√5) -2))⁄((2x)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2))) = (1/2+it)

**Input:**

$$\frac{\sqrt{10-2\sqrt{5}}-2}{2\,x}\times\frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}=\frac{1}{2}+i\,t$$

$i$ is the imaginary unit

**Exact result:**

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

**Alternate form assuming t and x are real:**

$$\frac{\sqrt{5}-1}{x}=2$$

## Alternate form:

$$\frac{\left(\sqrt{5}-1\right)(1+2\,i\,t)}{4\,x}=\frac{1}{2}+i\,t$$

## Alternate form assuming t and x are positive:

$$2\,x+1=\sqrt{5}$$

## Expanded forms:

$$\frac{i\sqrt{5\left(10-2\sqrt{5}\right)}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{10-2\sqrt{5}}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{5}\;t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{i\,t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{10-2\sqrt{5}}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{5}}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

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$$\frac{i\sqrt{5}\;t}{2\,x}-\frac{i\,t}{2\,x}+\frac{\sqrt{5}}{4\,x}-\frac{1}{4\,x}=\frac{1}{2}+i\,t$$

## Solutions:

$$t=\frac{i}{2}\,,\quad x\neq 0$$

$$x = \frac{\sqrt{5}}{2} - \frac{1}{2}$$

**Input:**

$$\frac{\sqrt{5}}{2} - \frac{1}{2}$$

**Decimal approximation:**

0.6180339887498948482045868343656381177203091798057628621354486227

…

0.6180339887…. $= \frac{1}{\phi}$

**Solution for the variable x:**

$$x = \frac{-2\, i \sqrt{5}\, t + 2\, i\, t - \sqrt{5} + 1}{-2 - 4\, i\, t}$$

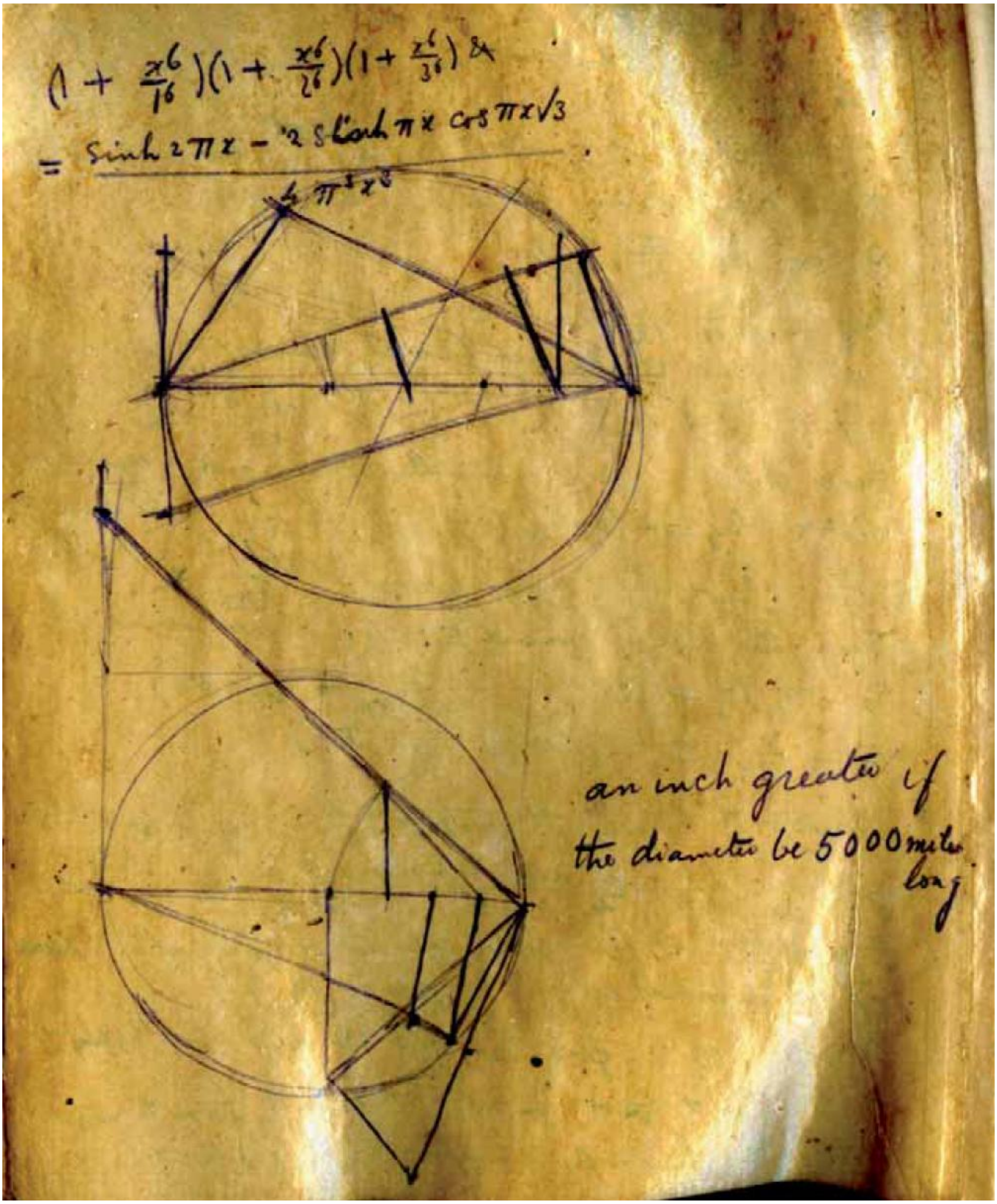
**Implicit derivatives:**

$$\frac{\partial x(t)}{\partial t} = \frac{2\left(-1 + \sqrt{5} - 2\, x\right) x}{\left(-1 + \sqrt{5}\right)(-i + 2\, t)}$$

$$\frac{\partial t(x)}{\partial x} = \frac{\left(-1 + \sqrt{5}\right)(-i + 2\, t)}{2\left(-1 + \sqrt{5} - 2\, x\right) x}$$

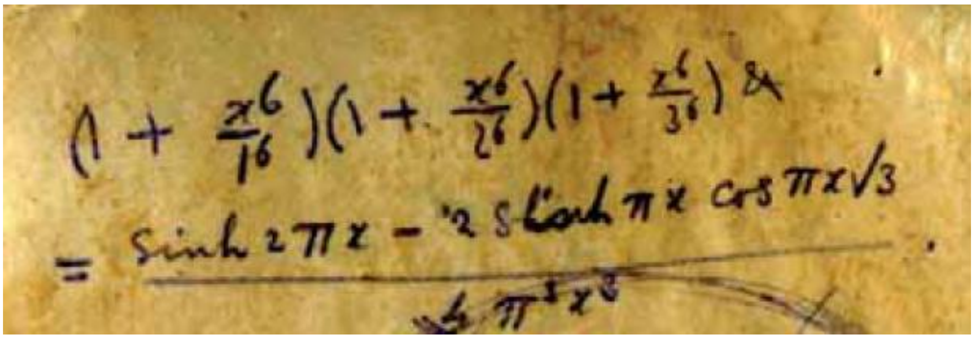
From:

**Manuscript Book 1** – *Srinivasa Ramanujan*



$$\left(1+\frac{x^6}{1^6}\right)\left(1+\frac{x^6}{2^6}\right)\left(1+\frac{x^6}{3^6}\right) \&c = \frac{\sinh 2\pi x - 2\cosh \pi x \cos \pi x\sqrt{3}}{4\pi^3 x^3}.$$

an inch greater if the diameter be 5000 miles long



$$\left(1+\frac{x^6}{1^6}\right)\left(1+\frac{x^6}{2^6}\right)\left(1+\frac{x^6}{3^6}\right)\&c = \frac{\sinh 2\pi x - 2\cosh \pi x \cos \pi x\sqrt{3}}{4\pi^3 x^3}.$$

(sinh (2Pi*x) – 2 tanh(Pi*x) cos(Pi*x*sqrt3)) / (4Pi^3*x^3)

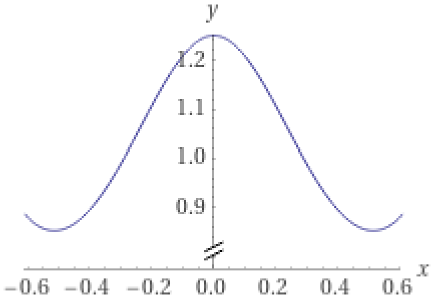
## Input

$$\frac{\sinh(2\pi x) - 2\tanh(\pi x)\cos\left(\pi x\sqrt{3}\right)}{4\pi^3 x^3}$$

$\sinh(x)$ is the hyperbolic sine function

$\tanh(x)$ is the hyperbolic tangent function

## Plots



(x from −0.6 to 0.6)

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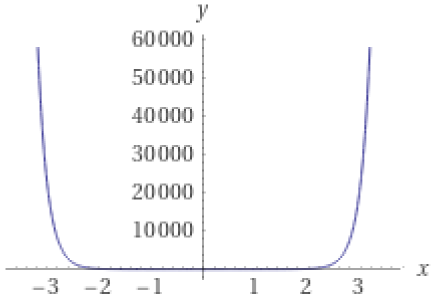

(x from −3.6 to 3.6)

## Alternate forms

$$-\frac{\tanh(\pi x)\left(2\cos\left(\sqrt{3}\,\pi x\right)-\cosh(2\pi x)-1\right)}{4\pi^3 x^3}$$

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$$\frac{\frac{\sinh(2\pi x)}{4\pi^3}-\frac{\cos\left(\sqrt{3}\,\pi x\right)\tanh(\pi x)}{2\pi^3}}{x^3}$$

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$$\frac{-\frac{2\cos\left(\sqrt{3}\,\pi x\right)\sinh(\pi x)}{\cosh(\pi x)}+\sinh(2\pi x)}{4\pi^3 x^3}$$

$\cosh(x)$ is the hyperbolic cosine function

## Expanded form

$$\frac{\sinh(2\pi x)}{4\pi^3 x^3}-\frac{\cos\left(\sqrt{3}\,\pi x\right)\tanh(\pi x)}{2\pi^3 x^3}$$

## Alternate form assuming x is real

$$\frac{\sinh(2\pi x)}{4\pi^3 x^3}-\frac{\cos\left(\sqrt{3}\,\pi x\right)\sinh(2\pi x)}{2\pi^3 x^3(\cosh(2\pi x)+1)}$$

## Series expansion at x=0

$$\frac{5}{4}-\frac{7\pi^2 x^2}{16}+\frac{103\pi^4 x^4}{480}+O\left(x^6\right)$$

(Taylor series)

## Derivative

$$\frac{d}{dx}\left(\frac{\sinh(2\pi x)-2\tanh(\pi x)\cos\left(\pi x\sqrt{3}\right)}{4\pi^3 x^3}\right)=\frac{1}{4\pi^3 x^4}$$
$$\left(-3\sinh(2\pi x)+2\pi x\cosh(2\pi x)+2\sqrt{3}\,\pi x\sin\left(\sqrt{3}\,\pi x\right)\tanh(\pi x)+\right.$$
$$\left.6\cos\left(\sqrt{3}\,\pi x\right)\tanh(\pi x)-2\pi x\cos\left(\sqrt{3}\,\pi x\right)\operatorname{sech}^2(\pi x)\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

For x = 2, we obtain:

(1+2^6)(1+2^6/2^6)(1+2^6/3^6)(1+2^6/4^6)(1+2^6/5^6)

**Input**

$$(1+2^6)\left(1+\frac{2^6}{2^6}\right)\left(1+\frac{2^6}{3^6}\right)\left(1+\frac{2^6}{4^6}\right)\left(1+\frac{2^6}{5^6}\right)$$

**Exact result**

$$\frac{2102592713}{14580000}$$

**Decimal approximation**

144.21074849108367626886145404663923182441700960219478737997256515

…

144.210748491….

(sinh (4Pi) – 2 tanh(2Pi) cos(2Pi*sqrt3)) / (4Pi^3*8)

**Input**

$$\frac{\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\times 8}$$

$\sinh(x)$ is the hyperbolic sine function

$\tanh(x)$ is the hyperbolic tangent function

**Exact result**

$$\frac{\sinh(4\pi)-2\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\pi)}{32\pi^3}$$

## Decimal approximation

144.50288074099753850898434343996996931241080108462749027394687888

…

144.50288074….

## Alternate forms

$$\frac{-\frac{2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\,\pi)}{\cosh(2\,\pi)}+\sinh(4\,\pi)}{32\,\pi^3}$$

$$\frac{\sinh(4\,\pi)}{32\,\pi^3}-\frac{\cos\left(2\sqrt{3}\,\pi\right)\sinh(4\,\pi)}{16\,\pi^3\,(1+\cosh(4\,\pi))}$$

$$\frac{\sinh(\pi)\cosh(\pi)\left(1-2\cos\left(2\sqrt{3}\,\pi\right)+\cosh(4\,\pi)\right)}{16\,\pi^3\,(\cosh(\pi)-i\sinh(\pi))\,(\cosh(\pi)+i\sinh(\pi))}$$

$\cosh(x)$ is the hyperbolic cosine function

## Expanded form

$$\frac{\sinh(4\,\pi)}{32\,\pi^3}-\frac{\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\,\pi)}{16\,\pi^3}$$

## Alternative representations

$$\frac{\sinh(4\,\pi)-2\tanh(2\,\pi)\cos\left(2\,\pi\sqrt{3}\right)}{4\,\pi^3\,8}=\frac{\frac{1}{2}\left(-e^{-4\,\pi}+e^{4\,\pi}\right)-2\cosh\left(-2\,i\,\pi\sqrt{3}\right)\left(-1+\frac{2}{1+e^{-4\,\pi}}\right)}{32\,\pi^3}$$

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{\frac{1}{2}\left(-e^{-4\pi} + e^{4\pi}\right) - \left(e^{-2i\pi\sqrt{3}} + e^{2i\pi\sqrt{3}}\right)\left(-1 + \frac{2}{1+e^{-4\pi}}\right)}{32\pi^3}$$

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$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{i\cos\left(\frac{\pi}{2} + 4i\pi\right) - \left(e^{-2i\pi\sqrt{3}} + e^{2i\pi\sqrt{3}}\right)\left(-1 + \frac{2}{1+e^{-4\pi}}\right)}{32\pi^3}$$

## Series representations

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{8\pi^3\sum_{k=0}^{\infty} 4^{-2+k}\pi^{-3+2k}\left(\frac{(-3)^k}{(2k)!} + \frac{2^{1+2k}\pi}{(1+2k)!}\right) + \sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_2}(-1)^{k_1}(2\pi)^{2k_2}q^{2k_1}}{(2k_2)!}}{8\pi^3}$$

for $q = e^{2\pi}$

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$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{\pi^{5/2}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{(-4)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} - 16\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_2}(2\pi)^{2k_2}}{(2k_2)!\left(17-4k_1+4k_1^2\right)}}{16\pi^4}$$

---

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!} + 32\pi^3\sum_{j=0}^{\infty}\frac{\left(\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)}{16\pi^{5/2}}}{32\pi^3} \quad \text{for } q = e^{2\pi}$$

## Integral representations

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{2\pi\int_0^1 \cosh(4\pi t)\,dt + \int_0^1\int_0^1 \cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\operatorname{sech}^2(2\pi t_1)\,dt_2\,dt_1}{16\pi^3}$$

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\,e^{-(3\pi^2)/s+s}\left(e^{(7\pi^2)/s}\,\pi - s\int_0^{2\pi}\operatorname{sech}^2(t)\,dt\right)}{32\pi^{7/2}\,s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \int_0^1\left(\frac{\cosh(4\pi t)}{8\pi^2} + \frac{i\operatorname{sech}^2(2\pi t)}{16\pi^{5/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt \quad \text{for } \gamma > 0$$

## Multiple-argument formulas

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{3\sinh\left(\frac{4\pi}{3}\right) + 4\sinh^3\left(\frac{4\pi}{3}\right) - \frac{4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}}{32\pi^3}$$

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{2\cosh(2\pi)\sinh(2\pi) - \frac{4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}}{32\pi^3}$$

$$\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,8} = \frac{2\cosh(2\pi)\sinh(2\pi) - \frac{4\left(1-2\sin^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}}{32\pi^3}$$

From which:

12(((sinh (4Pi) – 2 tanh(2Pi) cos(2Pi*sqrt3)) / (4Pi^3*8))) – 5

**Input**

$$12\times\frac{\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\times 8} - 5$$

sinh(*x*) is the hyperbolic sine function

tanh(*x*) is the hyperbolic tangent function

**Exact result**

$$\frac{3\left(\sinh(4\pi) - 2\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\pi)\right)}{8\pi^3} - 5$$

**Decimal approximation**

1729.0345688919704621078121212796396317489296130155298832873625466…

1729.034568……

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternate forms**

$$-\frac{40\pi^3 - 3\sinh(4\pi) + 6\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\pi)}{8\pi^3}$$

$$-5+\frac{3\left(-\frac{2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}{\cosh(2\pi)}+\sinh(4\pi)\right)}{8\pi^3}$$

$$-\frac{40\pi^3-3\sinh(4\pi)}{8\pi^3}-\frac{3\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\pi)}{4\pi^3}$$

$\cosh(x)$ is the hyperbolic cosine function

## Expanded form

$$-5+\frac{3\sinh(4\pi)}{8\pi^3}-\frac{3\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\pi)}{4\pi^3}$$

## Alternative representations

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=$$
$$-5+\frac{12\left(\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)-2\cosh\left(-2i\pi\sqrt{3}\right)\left(-1+\frac{2}{1+e^{-4\pi}}\right)\right)}{32\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=$$
$$-5+\frac{12\left(\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)-\left(e^{-2i\pi\sqrt{3}}+e^{2i\pi\sqrt{3}}\right)\left(-1+\frac{2}{1+e^{-4\pi}}\right)\right)}{32\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=$$
$$-5+\frac{12\left(i\cos\left(\frac{\pi}{2}+4i\pi\right)-\left(e^{-2i\pi\sqrt{3}}+e^{2i\pi\sqrt{3}}\right)\left(-1+\frac{2}{1+e^{-4\pi}}\right)\right)}{32\pi^3}$$

## Series representations

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=\frac{1}{2\pi^3}\left(-10\pi^3+2\pi^3\sum_{k=0}^{\infty}\frac{3\times 4^{-1+k}\,\pi^{-3+2k}\left(2^{1+2k}\,\pi\,(2k)!+(-3)^k\,(1+2k)!\right)}{(2k)!\,(1+2k)!}+3\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_2}\,(-1)^{k_1}\,(2\pi)^{2k_2}\,q^{2k_1}}{(2k_2)!}\right)\ \text{for } q=e^{2\pi}$$

---

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=-\frac{20\pi^4-3\pi^{5/2}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{(-4)^{-s}\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}+48\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_2}\,(2\pi)^{2k_2}}{(2k_2)!\left(17-4k_1+4k_1^2\right)}}{4\pi^4}$$

---

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=\frac{-40\pi^3+3\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}+8\pi^3\sum_{j=0}^{\infty}\frac{3\left(\operatorname{Res}_{s=-j}\frac{3^{-s}\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)\left(1+2\sum_{k=1}^{\infty}(-1)^k\,q^{2k}\right)}{4\pi^{5/2}}}{8\pi^3}\ \text{for } q=e^{2\pi}$$

## Integral representations

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=-\frac{20\pi^3-6\pi\int_0^1\cosh(4\pi t)\,dt+\int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\operatorname{sech}^2(2\pi t_1)\,dt_2\,dt_1}{4\pi^3}$$

---

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=$$

$$-5+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{3\,i\,e^{-(3\pi^2)/s+s}\left(e^{(7\pi^2)/s}\,\pi-s\int_0^{2\pi}\operatorname{sech}^2(t)\,dt\right)}{8\,\pi^{7/2}\,s^{3/2}}\,ds \quad \text{for } \gamma>0$$

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=$$

$$-5+\int_0^1\left(\frac{3\cosh(4\pi t)}{2\pi^2}+\frac{3\,i\operatorname{sech}^2(2\pi t)}{4\,\pi^{5/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt \quad \text{for } \gamma>0$$

## Multiple-argument formulas

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=$$

$$-5+\frac{3\left(3\sinh\left(\frac{4\pi}{3}\right)+4\sinh^3\left(\frac{4\pi}{3}\right)-\frac{4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}\right)}{8\,\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=$$

$$-5+\frac{3\left(2\cosh(2\pi)\sinh(2\pi)-\frac{4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}\right)}{8\,\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5=$$

$$-5+\frac{3\left(2\cosh(2\pi)\sinh(2\pi)-\frac{4\left(1-2\sin^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}\right)}{8\,\pi^3}$$

(((12(((sinh (4Pi) – 2 tanh(2Pi) cos(2Pi*sqrt3)) / (4Pi^3*8))) - 5)))^1/15

## Input

$$\sqrt[15]{12\times\frac{\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\times 8}-5}$$

$\sinh(x)$ is the hyperbolic sine function

$\tanh(x)$ is the hyperbolic tangent function

## Exact result

$$\sqrt[15]{\frac{3\left(\sinh(4\pi)-2\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\pi)\right)}{8\pi^3}-5}$$

## Decimal approximation

1.6438174197778730602515961719717346587291589932795269424141371749

...

1.6438174197……≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

## Alternate forms

$$\frac{\sqrt[15]{-40\pi^3+3\sinh(4\pi)-6\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\pi)}}{\sqrt[5]{2\pi}}$$

---

$$\sqrt[15]{-5+\frac{3\left(-\frac{2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}{\cosh(2\pi)}+\sinh(4\pi)\right)}{8\pi^3}}$$

---

$$\sqrt[15]{\frac{3\left(\sinh(4\pi)-\frac{2\cos\left(2\sqrt{3}\,\pi\right)\sinh(4\pi)}{1+\cosh(4\pi)}\right)}{8\pi^3}-5}$$

$\cosh(x)$ is the hyperbolic cosine function

## All 15th roots of (3 (sinh(4 π) - 2 cos(2 sqrt(3) π) tanh(2 π)))/(8 π^3) - 5

$$e^0 \sqrt[15]{\frac{3\left(\sinh(4\pi) - 2\cos\left(2\sqrt{3}\ \pi\right)\tanh(2\pi)\right)}{8\pi^3} - 5} \approx 1.6438 \text{ (real, principal root)}$$

---

$$e^{(2 i \pi)/15} \sqrt[15]{\frac{3\left(\sinh(4\pi) - 2\cos\left(2\sqrt{3}\ \pi\right)\tanh(2\pi)\right)}{8\pi^3} - 5} \approx 1.5017 + 0.6686\, i$$

---

$$e^{(4 i \pi)/15} \sqrt[15]{\frac{3\left(\sinh(4\pi) - 2\cos\left(2\sqrt{3}\ \pi\right)\tanh(2\pi)\right)}{8\pi^3} - 5} \approx 1.0999 + 1.2216\, i$$

---

$$e^{(2 i \pi)/5} \sqrt[15]{\frac{3\left(\sinh(4\pi) - 2\cos\left(2\sqrt{3}\ \pi\right)\tanh(2\pi)\right)}{8\pi^3} - 5} \approx 0.5080 + 1.5634\, i$$

---

$$e^{(8 i \pi)/15} \sqrt[15]{\frac{3\left(\sinh(4\pi) - 2\cos\left(2\sqrt{3}\ \pi\right)\tanh(2\pi)\right)}{8\pi^3} - 5} \approx -0.17183 + 1.6348\, i$$

---

## Alternative representations

$$\sqrt[15]{\frac{12\left(\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\, 8} - 5} =$$

$$\sqrt[15]{-5 + \frac{12\left(\frac{1}{2}\left(-e^{-4\pi} + e^{4\pi}\right) - 2\cosh\left(-2 i \pi \sqrt{3}\right)\left(-1 + \frac{2}{1+e^{-4\pi}}\right)\right)}{32\pi^3}}$$

$$\sqrt[15]{\frac{12\left(\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\, 8} - 5} = \sqrt[15]{-5 + \frac{12\left(\frac{1}{2}\left(-e^{-4\pi} + e^{4\pi}\right) - \left(e^{-2i\pi\sqrt{3}} + e^{2i\pi\sqrt{3}}\right)\left(-1 + \frac{2}{1+e^{-4\pi}}\right)\right)}{32\pi^3}}$$

$$\sqrt[15]{\frac{12\left(\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\, 8} - 5} = \sqrt[15]{-5 + \frac{12\left(i\cos\left(\frac{\pi}{2} + 4i\pi\right) - \left(e^{-2i\pi\sqrt{3}} + e^{2i\pi\sqrt{3}}\right)\left(-1 + \frac{2}{1+e^{-4\pi}}\right)\right)}{32\pi^3}}$$

## Series representations

$$\sqrt[15]{\frac{12\left(\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\, 8} - 5} = \frac{1}{\sqrt[5]{2\pi}}\left(\left(-40\pi^3 + 6\sum_{k=0}^{\infty}\frac{(-3)^k (2\pi)^{2k}}{(2k)!} + 3\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!} + 12\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_2}(-1)^{k_1}(2\pi)^{2k_2} q^{2k_1}}{(2k_2)!}\right) \hat{}\ (1/15)\right) \text{ for } q = e^{2\pi}$$

$$\sqrt[15]{\frac{12\left(\sinh(4\pi) - 2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\, 8} - 5} = \frac{\sqrt[15]{-20\pi^4 + 3\pi^{5/2}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{(-4)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} - 48\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_2}(2\pi)^{2k_2}}{(2k_2)!\left(17-4k_1+4k_1^2\right)}}}{2^{2/15}\pi^{4/15}}$$

$$\sqrt[15]{\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5}=\left(-5+\sum_{j=0}^{\infty}\frac{3\left(\pi^2\left(\operatorname{Res}_{s=-j}\frac{(-4)^{-s}\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)-16\left(\operatorname{Res}_{s=-j}\frac{3^{-s}\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)\sum_{k=1}^{\infty}\frac{1}{17-4k+4k^2}\right)}{4\pi^{7/2}}\right)\text{^}(1/15)$$

## Integral representations

$$\sqrt[15]{\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5}=\frac{1}{2^{2/15}\sqrt[5]{\pi}}\left(\left(-20\pi^3+6\pi\int_0^1\cosh(4\pi t)\,dt+\int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\operatorname{sech}^2(2\pi t_1)\,dt_2\,dt_1\right)\text{^}(1/15)\right)$$

---

$$\sqrt[15]{\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5}=\sqrt[15]{-5+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{3\,i\,e^{-(3\pi^2)/s+s}\left(e^{(7\pi^2)/s}\,\pi-s\int_0^{2\pi}\operatorname{sech}^2(t)\,dt\right)}{8\pi^{7/2}\,s^{3/2}}\,ds}\quad\text{for }\gamma>0$$

---

$$\sqrt[15]{\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5}=\sqrt[15]{-5+\int_0^1\left(\frac{3\cosh(4\pi t)}{2\pi^2}+\frac{3\,i\operatorname{sech}^2(2\pi t)}{4\pi^{5/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt}\quad\text{for }\gamma>0$$

## Multiple-argument formulas

$$\sqrt[15]{\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5} = \sqrt[15]{-5+\frac{3\left(2\cosh(2\pi)\sinh(2\pi)-\frac{4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}\right)}{8\pi^3}}$$

---

$$\sqrt[15]{\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5} = \sqrt[15]{-5+\frac{3\left(2\cosh(2\pi)\sinh(2\pi)-\frac{4\left(1-2\sin^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}\right)}{8\pi^3}}$$

---

$$\sqrt[15]{\frac{12\left(\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,8}-5} = \sqrt[15]{-5+\frac{3\left(3\sinh\left(\frac{4\pi}{3}\right)+4\sinh^3\left(\frac{4\pi}{3}\right)-\frac{4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\tanh(\pi)}{1+\tanh^2(\pi)}\right)}{8\pi^3}}$$

And:

(1/27((((12(((sinh (4Pi) – 2 tanh(2Pi) cos(2Pi*sqrt3)) / (4Pi^3*8))) - 5)))-1))^2-(((√(10-2√5) -2))⁄((√5-1)))^2

## Input

$$\left(\frac{1}{27}\left(\left(12\times\frac{\sinh(4\pi)-2\tanh(2\pi)\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\times 8}-5\right)-1\right)\right)^2-\left(\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)^2$$

sinh($x$) is the hyperbolic sine function tanh($x$) is the hyperbolic tangent function

**Exact result**

$$\frac{1}{729}\left(\frac{3\left(\sinh(4\pi)-2\cos\left(2\sqrt{3}\,\pi\right)\tanh(2\pi)\right)}{8\pi^3}-6\right)^2-\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)^2}{\left(\sqrt{5}-1\right)^2}$$

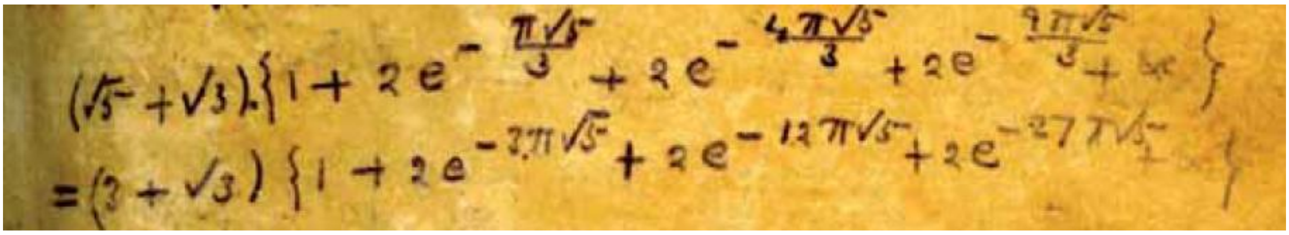
**Decimal approximation**

4096.0831828906205512789348123037475581663521784150054485348048478

...

4096.08318….≈ 4096 = $64^2$

From:



$$(\sqrt{5}+\sqrt{3})\left\{1+2e^{-\frac{\pi\sqrt{5}}{3}}+2e^{-\frac{4\pi\sqrt{5}}{3}}+2e^{-\frac{9\pi\sqrt{5}}{3}}+\&c\right\}$$
$$=(3+\sqrt{3})\left\{1+2e^{-3\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}+\&c\right\}$$

(sqrt5+sqrt3) [1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)]

**Input**

$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times4\left(\pi\sqrt{5}\right)}+2e^{-1/3\times9\left(\pi\sqrt{5}\right)}\right)$$

**Exact result**

$$\left(\sqrt{3}+\sqrt{5}\right)\left(1+2e^{-3\sqrt{5}\,\pi}+2e^{-(4\sqrt{5}\,\pi)/3}+2e^{-(\sqrt{5}\,\pi)/3}\right)$$

**Decimal approximation**

4.7320508142302624451212874346583002327722603647641713992087439108

...

4.73205081423….

## Alternate forms

$$\left(\sqrt{3}+\sqrt{5}\right)\left(1+2\,e^{-3\sqrt{5}\,\pi}\left(1+e^{(5\sqrt{5}\,\pi)/3}+e^{(8\sqrt{5}\,\pi)/3}\right)\right)$$

$$\left(\sqrt{3}+\sqrt{5}\right)e^{-3\sqrt{5}\,\pi}\left(2+2\,e^{(5\sqrt{5}\,\pi)/3}+2\,e^{(8\sqrt{5}\,\pi)/3}+e^{3\sqrt{5}\,\pi}\right)$$

$$\sqrt{3}+\sqrt{5}+2\left(\sqrt{3}+\sqrt{5}\right)e^{-3\sqrt{5}\,\pi}+\\ 2\left(\sqrt{3}+\sqrt{5}\right)e^{-(4\sqrt{5}\,\pi)/3}+2\left(\sqrt{3}+\sqrt{5}\right)e^{-(\sqrt{5}\,\pi)/3}$$

## Expanded form

$$\sqrt{3}+\sqrt{5}+2\sqrt{3}\;e^{-3\sqrt{5}\,\pi}+2\sqrt{5}\;e^{-3\sqrt{5}\,\pi}+2\sqrt{3}\;e^{-(4\sqrt{5}\,\pi)/3}+\\ 2\sqrt{5}\;e^{-(4\sqrt{5}\,\pi)/3}+2\sqrt{3}\;e^{-(\sqrt{5}\,\pi)/3}+2\sqrt{5}\;e^{-(\sqrt{5}\,\pi)/3}$$

## Series representations

$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3(\pi\sqrt{5})(-1)}+2\,e^{1/3(4(\pi\sqrt{5}))(-1)}+2\,e^{1/3(9(\pi\sqrt{5}))(-1)}\right)=\\ e^{-3\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\\ \left(2+2\,e^{5/3\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+2\,e^{8/3\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+e^{3\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\right)\\ \sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)$$

$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)=$$

$$\exp\left(-3\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\left(2+2\exp\left(\frac{5}{3}\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)+2\exp\left(\frac{8}{3}\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)+\right.$$

$$\left.\exp\left(3\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_{k}\left(\left(-\frac{1}{2}\right)^{k}\sqrt{2}+\left(-\frac{1}{4}\right)^{k}\sqrt{4}\right)}{k!}$$

---

$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)=$$

$$\exp\left(-3\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}\,z_0^{-k}}{k!}\right)$$

$$\left(2+2\exp\left(\frac{5}{3}\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}\,z_0^{-k}}{k!}\right)+\right.$$

$$2\exp\left(\frac{8}{3}\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}\,z_0^{-k}}{k!}\right)+$$

$$\left.\exp\left(3\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}\,z_0^{-k}}{k!}\right)\right)$$

$$\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left((3-z_0)^{k}+(5-z_0)^{k}\right)z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

(3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)]

**Input**

$$\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)$$

## Decimal approximation

4.7320582962070344203358871508574881888314709819745983166753686081

…

4.7320582962….

## Alternate forms

$$\left(3+\sqrt{3}\right)\left(1+2\,e^{-27\sqrt{5}\,\pi}\left(1+e^{15\sqrt{5}\,\pi}+e^{25\sqrt{5}\,\pi}\right)\right)$$

$$\left(3+\sqrt{3}\right)e^{-27\sqrt{5}\,\pi}\left(2+2\,e^{15\sqrt{5}\,\pi}+2\,e^{25\sqrt{5}\,\pi}+e^{27\sqrt{5}\,\pi}\right)$$

$$3+\sqrt{3}+2\left(3+\sqrt{3}\right)e^{-27\sqrt{5}\,\pi}+2\left(3+\sqrt{3}\right)e^{-12\sqrt{5}\,\pi}+2\left(3+\sqrt{3}\right)e^{-2\sqrt{5}\,\pi}$$

## Expanded form

$$3+\sqrt{3}+6\,e^{-27\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-27\sqrt{5}\,\pi}+6\,e^{-12\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-12\sqrt{5}\,\pi}+6\,e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-2\sqrt{5}\,\pi}$$

## Series representations

$$\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)=e^{-27\pi\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\left(2+2\,e^{15\pi\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+2\,e^{25\pi\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+e^{27\pi\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\right)\left(3+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$(3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)=$$
$$\exp\left(-27\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\left(2+2\exp\left(15\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+2\exp\left(25\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+\right.$$
$$\left.\exp\left(27\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\left(3+\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

---

$$(3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)=$$
$$\exp\left(-27\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)$$
$$\left(2+2\exp\left(15\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)+\right.$$
$$2\exp\left(25\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)+$$
$$\left.\exp\left(27\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$
$$\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\quad \text{for (not } (z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$$

From the sum of the two expressions, adding

$$\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}\approx 0.46988470906324$$

we obtain:

(3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)] + (sqrt5+sqrt3) [1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)]+(((2 log^(7/8)(9))/(3^(1/4) e^2 log^(3/8)(2))))

## Input

$$(3+\sqrt{3})\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+$$
$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{-(\pi\sqrt{5})/3}+2\,e^{-1/3\times 4(\pi\sqrt{5})}+2\,e^{-1/3\times 9(\pi\sqrt{5})}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\;e^2\log^{3/8}(2)}$$

log($x$) is the natural logarithm

## Exact result

$$(3+\sqrt{3})\left(1+2\,e^{-27\sqrt{5}\,\pi}+2\,e^{-12\sqrt{5}\,\pi}+2\,e^{-2\sqrt{5}\,\pi}\right)+$$
$$\left(\sqrt{3}+\sqrt{5}\right)\left(1+2\,e^{-3\sqrt{5}\,\pi}+2\,e^{-(4\sqrt{5}\,\pi)/3}+2\,e^{-(\sqrt{5}\,\pi)/3}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\;e^2\log^{3/8}(2)}$$

## Decimal approximation

9.9339938195005454861512532344337251717479359749022354303432581939

…

9.9339938195….result very near to the entropy black hole ln(20619) = 9.933968….

## Alternate forms

$$3+2\sqrt{3}+\sqrt{5}+(6+2\sqrt{3})\,e^{-27\sqrt{5}\,\pi}+(6+2\sqrt{3})\,e^{-12\sqrt{5}\,\pi}+$$
$$(2\sqrt{3}+2\sqrt{5})\,e^{-3\sqrt{5}\,\pi}+(6+2\sqrt{3})\,e^{-2\sqrt{5}\,\pi}+$$
$$(2\sqrt{3}+2\sqrt{5})\,e^{-(4\sqrt{5}\,\pi)/3}+(2\sqrt{3}+2\sqrt{5})\,e^{-(\sqrt{5}\,\pi)/3}+\frac{2\,(2\log(3))^{7/8}}{\sqrt[4]{3}\;e^2\log^{3/8}(2)}$$

---

$$2(3+\sqrt{3})\,e^{-27\sqrt{5}\,\pi}+2(3+\sqrt{3})\,e^{-12\sqrt{5}\,\pi}+2(\sqrt{3}+\sqrt{5})\,e^{-3\sqrt{5}\,\pi}+$$
$$2(3+\sqrt{3})\,e^{-2\sqrt{5}\,\pi}+2(\sqrt{3}+\sqrt{5})\,e^{-(4\sqrt{5}\,\pi)/3}+2(\sqrt{3}+\sqrt{5})\,e^{-(\sqrt{5}\,\pi)/3}+$$
$$\frac{2\times 3^{3/4}\log^{7/8}(9)+9\,e^2\log^{3/8}(2)+6\sqrt{3}\;e^2\log^{3/8}(2)+3\sqrt{5}\;e^2\log^{3/8}(2)}{3\,e^2\log^{3/8}(2)}$$

---

$$\frac{1}{\sqrt[4]{3}\; e^2 \log^{3/8}(2)} \Big(\sqrt[4]{3}\,\left(3+2\sqrt{3}+\sqrt{5}\right) e^2 \log^{3/8}(2) + 2\sqrt[4]{3}\,\left(3+\sqrt{3}\right) e^{2-27\sqrt{5}\,\pi} \log^{3/8}(2) +$$
$$2\sqrt[4]{3}\,\left(3+\sqrt{3}\right) e^{2-12\sqrt{5}\,\pi} \log^{3/8}(2) + 2\sqrt[4]{3}\,\left(\sqrt{3}+\sqrt{5}\right) e^{2-3\sqrt{5}\,\pi} \log^{3/8}(2) +$$
$$2\sqrt[4]{3}\,\left(3+\sqrt{3}\right) e^{2-2\sqrt{5}\,\pi} \log^{3/8}(2) + 2\sqrt[4]{3}\,\left(\sqrt{3}+\sqrt{5}\right) e^{2-(4\sqrt{5}\,\pi)/3} \log^{3/8}(2) +$$
$$2\sqrt[4]{3}\,\left(\sqrt{3}+\sqrt{5}\right) e^{2-(\sqrt{5}\,\pi)/3} \log^{3/8}(2) + 2\,(2\log(3))^{7/8}\Big)$$

## Expanded form

$$3+2\sqrt{3}+\sqrt{5}+6\,e^{-27\sqrt{5}\,\pi}+2\sqrt{3}\;e^{-27\sqrt{5}\,\pi}+6\,e^{-12\sqrt{5}\,\pi}+2\sqrt{3}\;e^{-12\sqrt{5}\,\pi}+$$
$$2\sqrt{3}\;e^{-3\sqrt{5}\,\pi}+2\sqrt{5}\;e^{-3\sqrt{5}\,\pi}+6\,e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\;e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\;e^{-(4\sqrt{5}\,\pi)/3}+$$
$$2\sqrt{5}\;e^{-(4\sqrt{5}\,\pi)/3}+2\sqrt{3}\;e^{-(\sqrt{5}\,\pi)/3}+2\sqrt{5}\;e^{-(\sqrt{5}\,\pi)/3}+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\;e^2\log^{3/8}(2)}$$

## Alternative representations

$$\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+$$
$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+$$
$$\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\;e^2\log^{3/8}(2)} =$$
$$\frac{2\log_e^{7/8}(9)}{\sqrt[4]{3}\;e^2\log_e^{3/8}(2)}+\left(1+2\,e^{-27\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-2\pi\sqrt{5}}\right)\left(3+\sqrt{3}\right)+$$
$$\left(1+2\,e^{-3\pi\sqrt{5}}+2\,e^{-(4\pi\sqrt{5})/3}+2\,e^{-(\pi\sqrt{5})/3}\right)\left(\sqrt{3}+\sqrt{5}\right)$$

---

$$\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+$$
$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+$$
$$\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\;e^2\log^{3/8}(2)} = \frac{2\left(\log(a)\log_a(9)\right)^{7/8}}{\sqrt[4]{3}\;e^2\left(\log(a)\log_a(2)\right)^{3/8}}+$$
$$\left(1+2\,e^{-27\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-2\pi\sqrt{5}}\right)\left(3+\sqrt{3}\right)+$$
$$\left(1+2\,e^{-3\pi\sqrt{5}}+2\,e^{-(4\pi\sqrt{5})/3}+2\,e^{-(\pi\sqrt{5})/3}\right)\left(\sqrt{3}+\sqrt{5}\right)$$

$$\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}=$$
$$\frac{2\left(-\mathrm{Li}_1(-8)\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(-\mathrm{Li}_1(-1)\right)^{3/8}}+\left(1+2\,e^{-27\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-2\pi\sqrt{5}}\right)\left(3+\sqrt{3}\right)+\left(1+2\,e^{-3\pi\sqrt{5}}+2\,e^{-\left(4\pi\sqrt{5}\right)/3}+2\,e^{-\left(\pi\sqrt{5}\right)/3}\right)\left(\sqrt{3}+\sqrt{5}\right)$$

## Series representations

$$\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}=3+2\sqrt{3}+\sqrt{5}+6\,e^{-27\sqrt{5}\,\pi}+$$
$$2\sqrt{3}\,e^{-27\sqrt{5}\,\pi}+6\,e^{-12\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-12\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-3\sqrt{5}\,\pi}+$$
$$2\sqrt{5}\,e^{-3\sqrt{5}\,\pi}+6\,e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-\left(4\sqrt{5}\,\pi\right)/3}+$$
$$2\sqrt{5}\,e^{-\left(4\sqrt{5}\,\pi\right)/3}+2\sqrt{3}\,e^{-\left(\sqrt{5}\,\pi\right)/3}+2\sqrt{5}\,e^{-\left(\sqrt{5}\,\pi\right)/3}+$$
$$\frac{2\left(2\,i\,\pi\left\lfloor\frac{\arg(9-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(9-x)^k\,x^{-k}}{k}\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(2\,i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}\right)^{3/8}}\quad\text{for } x<0$$

$$\begin{aligned}&\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+\\&\quad\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+\\&\quad\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}=3+2\sqrt{3}+\sqrt{5}+6\,e^{-27\sqrt{5}\,\pi}+\\&\quad 2\sqrt{3}\,e^{-27\sqrt{5}\,\pi}+6\,e^{-12\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-12\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-3\sqrt{5}\,\pi}+\\&\quad 2\sqrt{5}\,e^{-3\sqrt{5}\,\pi}+6\,e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-\left(4\sqrt{5}\,\pi\right)/3}+\\&\quad 2\sqrt{5}\,e^{-\left(4\sqrt{5}\,\pi\right)/3}+2\sqrt{3}\,e^{-\left(\sqrt{5}\,\pi\right)/3}+2\sqrt{5}\,e^{-\left(\sqrt{5}\,\pi\right)/3}+\\&\quad\frac{2\left(\log(z_0)+\left\lfloor\frac{\arg(9-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(9-z_0)^k z_0^{-k}}{k}\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{3/8}}\end{aligned}$$

---

$$\begin{aligned}&\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+\\&\quad\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+\\&\quad\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}=3+2\sqrt{3}+\sqrt{5}+6\,e^{-27\sqrt{5}\,\pi}+\\&\quad 2\sqrt{3}\,e^{-27\sqrt{5}\,\pi}+6\,e^{-12\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-12\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-3\sqrt{5}\,\pi}+\\&\quad 2\sqrt{5}\,e^{-3\sqrt{5}\,\pi}+6\,e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-2\sqrt{5}\,\pi}+2\sqrt{3}\,e^{-\left(4\sqrt{5}\,\pi\right)/3}+\\&\quad 2\sqrt{5}\,e^{-\left(4\sqrt{5}\,\pi\right)/3}+2\sqrt{3}\,e^{-\left(\sqrt{5}\,\pi\right)/3}+2\sqrt{5}\,e^{-\left(\sqrt{5}\,\pi\right)/3}+\\&\quad\frac{2\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(9-z_0)^k z_0^{-k}}{k}\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{3/8}}\end{aligned}$$

Indeed:

exp((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)] + (sqrt5+sqrt3) [1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)]+(((2 log^(7/8)(9))/(3^(1/4) e^2 log^(3/8)(2)))))

## Input

$$\exp\left((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+(\sqrt{5}+\sqrt{3})\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4(\pi\sqrt{5})}+2e^{-1/3\times 9(\pi\sqrt{5})}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\;e^2\log^{3/8}(2)}\right)$$

log(*x*) is the natural logarithm

## Decimal approximation

20619.527022412857315359405911683176348778835603237938249003761535

…

**20619**.527022

## Alternate form

$$e^{3+2\sqrt{3}+\sqrt{5}+(6+2\sqrt{3})e^{-27\sqrt{5}\pi}+(6+2\sqrt{3})e^{-12\sqrt{5}\pi}+(2\sqrt{3}+2\sqrt{5})e^{-3\sqrt{5}\pi}+(6+2\sqrt{3})e^{-2\sqrt{5}\pi}+(2\sqrt{3}+2\sqrt{5})e^{-\left(4\sqrt{5}\pi\right)/3}+(2\sqrt{3}+2\sqrt{5})e^{-\left(\sqrt{5}\pi\right)/3}+\frac{2(2\log(3))^{7/8}}{\sqrt[4]{3}\;e^2\log^{3/8}(2)}}$$

## Alternative representations

$$\exp\left((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+(\sqrt{5}+\sqrt{3})\left(1+2e^{1/3(\pi\sqrt{5})(-1)}+2e^{1/3(4(\pi\sqrt{5}))(-1)}+2e^{1/3(9(\pi\sqrt{5}))(-1)}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\;e^2\log^{3/8}(2)}\right)=$$
$$\exp\left(\frac{2\log_e^{7/8}(9)}{\sqrt[4]{3}\;e^2\log_e^{3/8}(2)}+\left(1+2e^{-27\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-2\pi\sqrt{5}}\right)(3+\sqrt{3})+\left(1+2e^{-3\pi\sqrt{5}}+2e^{-(4\pi\sqrt{5})/3}+2e^{-(\pi\sqrt{5})/3}\right)(\sqrt{3}+\sqrt{5})\right)$$

---

$$\exp\left(\left(3+\sqrt{3}\right)\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}\right)=\exp\left(\frac{2\left(\log(a)\log_a(9)\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(\log(a)\log_a(2)\right)^{3/8}}+\left(1+2e^{-27\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-2\pi\sqrt{5}}\right)\left(3+\sqrt{3}\right)+\left(1+2e^{-3\pi\sqrt{5}}+2e^{-\left(4\pi\sqrt{5}\right)/3}+2e^{-\left(\pi\sqrt{5}\right)/3}\right)\left(\sqrt{3}+\sqrt{5}\right)\right)$$

---

$$\exp\left(\left(3+\sqrt{3}\right)\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}\right)=\exp\left(\frac{2\left(-\mathrm{Li}_1(-8)\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(-\mathrm{Li}_1(-1)\right)^{3/8}}+\left(1+2e^{-27\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-2\pi\sqrt{5}}\right)\left(3+\sqrt{3}\right)+\left(1+2e^{-3\pi\sqrt{5}}+2e^{-\left(4\pi\sqrt{5}\right)/3}+2e^{-\left(\pi\sqrt{5}\right)/3}\right)\left(\sqrt{3}+\sqrt{5}\right)\right)$$

## Series representations

$$\exp\left(\left(3+\sqrt{3}\right)\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)+\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}\right)=\exp\left(\left(3+\sqrt{3}\right)\left(1+2e^{-27\sqrt{5}\,\pi}+2e^{-12\sqrt{5}\,\pi}+2e^{-2\sqrt{5}\,\pi}\right)+\left(\sqrt{3}+\sqrt{5}\right)\left(1+2e^{-3\sqrt{5}\,\pi}+2e^{-\left(4\sqrt{5}\,\pi\right)/3}+2e^{-\left(\sqrt{5}\,\pi\right)/3}\right)+\frac{2\left(2i\pi\left\lfloor\frac{\arg(9-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(9-x)^k x^{-k}}{k}\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^{3/8}}\right)\quad\text{for } x<0$$

---

$$\exp\left((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\right.$$
$$(\sqrt{5}+\sqrt{3})\left(1+2e^{1/3(\pi\sqrt{5})(-1)}+2e^{1/3(4(\pi\sqrt{5}))(-1)}+2e^{1/3(9(\pi\sqrt{5}))(-1)}\right)+$$
$$\left.\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}\right)=\exp\left((3+\sqrt{3})\left(1+2e^{-27\sqrt{5}\,\pi}+2e^{-12\sqrt{5}\,\pi}+2e^{-2\sqrt{5}\,\pi}\right)+\right.$$
$$(\sqrt{3}+\sqrt{5})\left(1+2e^{-3\sqrt{5}\,\pi}+2e^{-(4\sqrt{5}\,\pi)/3}+2e^{-(\sqrt{5}\,\pi)/3}\right)+$$
$$\left.\frac{2\left(\log(z_0)+\left\lfloor\frac{\arg(9-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(9-z_0)^k z_0^{-k}}{k}\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{3/8}}\right)$$

---

$$\exp\left((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\right.$$
$$(\sqrt{5}+\sqrt{3})\left(1+2e^{1/3(\pi\sqrt{5})(-1)}+2e^{1/3(4(\pi\sqrt{5}))(-1)}+2e^{1/3(9(\pi\sqrt{5}))(-1)}\right)+$$
$$\left.\frac{2\log^{7/8}(9)}{\sqrt[4]{3}\,e^2\log^{3/8}(2)}\right)=\exp\left((3+\sqrt{3})\left(1+2e^{-27\sqrt{5}\,\pi}+2e^{-12\sqrt{5}\,\pi}+2e^{-2\sqrt{5}\,\pi}\right)+\right.$$
$$(\sqrt{3}+\sqrt{5})\left(1+2e^{-3\sqrt{5}\,\pi}+2e^{-(4\sqrt{5}\,\pi)/3}+2e^{-(\sqrt{5}\,\pi)/3}\right)+$$
$$\left.\frac{2\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(9-z_0)^k z_0^{-k}}{k}\right)^{7/8}}{\sqrt[4]{3}\,e^2\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{3/8}}\right)$$

From the sum, after some calculations, we obtain also:

10^3+((((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)] + (sqrt5+sqrt3) [1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)]))-46/100)^3-1

**Input**

$$10^3+\left(\left((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times4(\pi\sqrt{5})}+2e^{-1/3\times9(\pi\sqrt{5})}\right)\right)-\frac{46}{100}\right)^3-1$$

**Exact result**

$$999+\left(-\frac{23}{50}+(3+\sqrt{3})\left(1+2e^{-27\sqrt{5}\,\pi}+2e^{-12\sqrt{5}\,\pi}+2e^{-2\sqrt{5}\,\pi}\right)+\left(\sqrt{3}+\sqrt{5}\right)\left(1+2e^{-3\sqrt{5}\,\pi}+2e^{-(4\sqrt{5}\,\pi)/3}+2e^{-(\sqrt{5}\,\pi)/3}\right)\right)^3$$

**Decimal approximation**

1728.9989697949364186709698477751093986364274383621166341262852193
...

1728.99896979…..

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2$ * $3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

((10^3+((((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)] + (sqrt5+sqrt3) [1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)]))-46/100)^3-1))^1/15

**Input**

$$\left(10^3+\left(\left((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times4(\pi\sqrt{5})}+2e^{-1/3\times9(\pi\sqrt{5})}\right)\right)-\frac{46}{100}\right)^3-1\right)\text{^}(1/15)$$

**Exact result**

$$\left(999+\left(-\frac{23}{50}+(3+\sqrt{3})\left(1+2e^{-27\sqrt{5}\,\pi}+2e^{-12\sqrt{5}\,\pi}+2e^{-2\sqrt{5}\,\pi}\right)+\left(\sqrt{3}+\sqrt{5}\right)\left(1+2e^{-3\sqrt{5}\,\pi}+2e^{-(4\sqrt{5}\,\pi)/3}+2e^{-(\sqrt{5}\,\pi)/3}\right)\right)^3\right)\text{^}(1/15)$$

**Decimal approximation**

1.6438151634521311330368338498898370457686383623229235349862962260

…

1.643815163452….≈ $\zeta(2) = \frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

(1/27((10^3+((((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)] + (sqrt5+sqrt3) [1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)]))-46/100)^3-2)))^2

**Input**

$$\left(\frac{1}{27}\left(10^3+\left(\left(\left(3+\sqrt{3}\right)\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4(\pi\sqrt{5})}+2e^{-1/3\times 9(\pi\sqrt{5})}\right)\right)-\frac{46}{100}\right)^3-2\right)\right)^2$$

**Exact result**

$$\frac{1}{729}\left(998+\left(-\frac{23}{50}+\left(3+\sqrt{3}\right)\left(1+2e^{-27\sqrt{5}\,\pi}+2e^{-12\sqrt{5}\,\pi}+2e^{-2\sqrt{5}\,\pi}\right)+\left(\sqrt{3}+\sqrt{5}\right)\left(1+2e^{-3\sqrt{5}\,\pi}+2e^{-(4\sqrt{5}\,\pi)/3}+2e^{-(\sqrt{5}\,\pi)/3}\right)\right)^3\right)^2$$

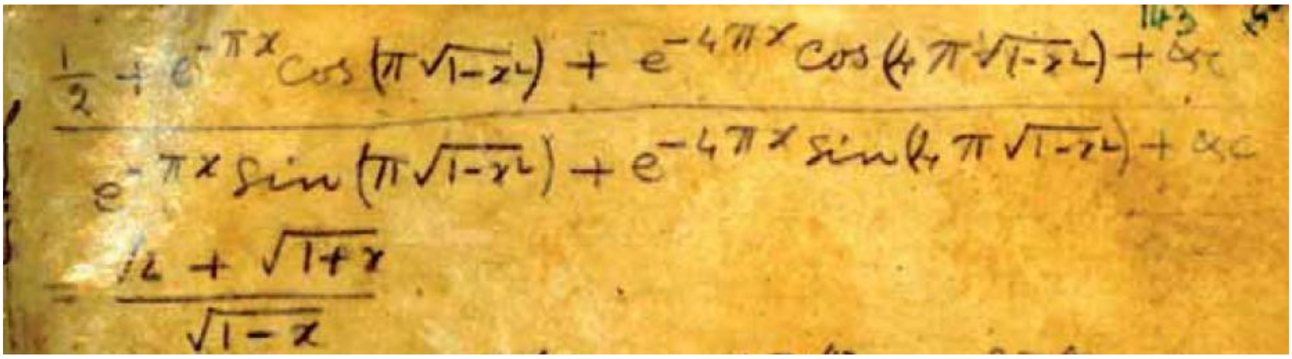
**Decimal approximation**

4095.9951160663396233194792923187646483056450550203046474848035000

…

4095.995116……≈ 4096 = $64^2$

From:



143 5

$$\frac{\frac{1}{2}+e^{-\pi x}\cos(\pi\sqrt{1-x^2})+e^{-4\pi x}\cos(4\pi\sqrt{1-x^2})+\&c}{e^{-\pi x}\sin(\pi\sqrt{1-x^2})+e^{-4\pi x}\sin(4\pi\sqrt{1-x^2})+\&c}=\frac{/2+\sqrt{1+x}}{\sqrt{1-x}}$$

[1/2+e^(-x*Pi) cos(Pi*sqrt(1-x^2))+e^(-4Pi*x) cos(4Pi*sqrt(1-x^2))] / [e^(-x*Pi) sin(Pi*sqrt(1-x^2)) +e^(-4Pi*x) sin(4Pi*sqrt(1-x^2))]

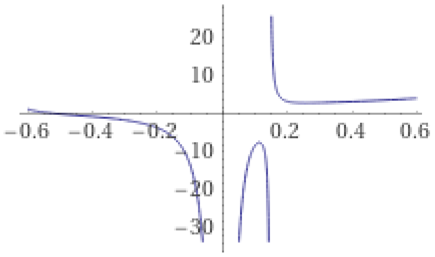
**Input**

$$\frac{\frac{1}{2}+e^{-x\pi}\cos\left(\pi\sqrt{1-x^2}\right)+e^{-4\pi x}\cos\left(4\pi\sqrt{1-x^2}\right)}{e^{-x\pi}\sin\left(\pi\sqrt{1-x^2}\right)+e^{-4\pi x}\sin\left(4\pi\sqrt{1-x^2}\right)}$$

**Exact result**

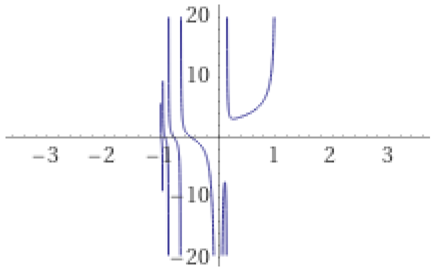
$$\frac{e^{-\pi x}\cos\left(\pi\sqrt{1-x^2}\right)+e^{-4\pi x}\cos\left(4\pi\sqrt{1-x^2}\right)+\frac{1}{2}}{e^{-\pi x}\sin\left(\pi\sqrt{1-x^2}\right)+e^{-4\pi x}\sin\left(4\pi\sqrt{1-x^2}\right)}$$

## Plots (figures that can be related to the open strings)



(x from −0.6 to 0.6)

---



(x from −3.6 to 3.6)

## Alternate forms

$$\frac{2 e^{3\pi x}\cos\left(\pi\sqrt{1-x^2}\right)+2\cos\left(4\pi\sqrt{1-x^2}\right)+e^{4\pi x}}{2\left(e^{3\pi x}\sin\left(\pi\sqrt{1-x^2}\right)+\sin\left(4\pi\sqrt{1-x^2}\right)\right)}$$

---

$$\frac{\left(2 e^{3\pi x}\cos\left(\pi\sqrt{1-x^2}\right)+2\cos\left(4\pi\sqrt{1-x^2}\right)+e^{4\pi x}\right)\csc\left(\pi\sqrt{1-x^2}\right)}{2\left(2\left(\cos\left(\pi\sqrt{1-x^2}\right)+\cos\left(3\pi\sqrt{1-x^2}\right)\right)+e^{3\pi x}\right)}$$

---

$$\frac{e^{-\pi x}\cos(\pi\sqrt{1-x}\ \sqrt{x+1}) + e^{-4\pi x}\cos(4\pi\sqrt{1-x}\ \sqrt{x+1}) + \frac{1}{2}}{e^{-\pi x}\sin(\pi\sqrt{1-x}\ \sqrt{x+1}) + e^{-4\pi x}\sin(4\pi\sqrt{1-x}\ \sqrt{x+1})}$$

$\csc(x)$ is the cosecant function

## Expanded form

$$\frac{1}{2\left(e^{-\pi x}\sin\left(\pi\sqrt{1-x^2}\right) + e^{-4\pi x}\sin\left(4\pi\sqrt{1-x^2}\right)\right)} + \frac{e^{-\pi x}\cos\left(\pi\sqrt{1-x^2}\right)}{e^{-\pi x}\sin\left(\pi\sqrt{1-x^2}\right) + e^{-4\pi x}\sin\left(4\pi\sqrt{1-x^2}\right)} + \frac{e^{-4\pi x}\cos\left(4\pi\sqrt{1-x^2}\right)}{e^{-\pi x}\sin\left(\pi\sqrt{1-x^2}\right) + e^{-4\pi x}\sin\left(4\pi\sqrt{1-x^2}\right)}$$

## Derivative

$$\begin{aligned}
&\frac{d}{dx}\left(\frac{\frac{1}{2} + e^{-x\pi}\cos\left(\pi\sqrt{1-x^2}\right) + e^{-4\pi x}\cos\left(4\pi\sqrt{1-x^2}\right)}{e^{-x\pi}\sin\left(\pi\sqrt{1-x^2}\right) + e^{-4\pi x}\sin\left(4\pi\sqrt{1-x^2}\right)}\right) = \\
&\Bigg(\pi\Bigg(\left(e^{3\pi x}\sin\left(\pi\sqrt{1-x^2}\right) + 4\sin\left(4\pi\sqrt{1-x^2}\right)\right) \\
&\qquad\left(e^{4\pi x}\sqrt{1-x^2} + 2e^{3\pi x}x\sin\left(\pi\sqrt{1-x^2}\right) + 2x\sin\left(4\pi\sqrt{1-x^2}\right)\right) + \\
&\quad 2e^{6\pi x}x\cos^2\left(\pi\sqrt{1-x^2}\right) + 8x\cos^2\left(4\pi\sqrt{1-x^2}\right) + \\
&\quad e^{3\pi x}\cos\left(\pi\sqrt{1-x^2}\right) \\
&\qquad\left(6\sqrt{1-x^2}\sin\left(4\pi\sqrt{1-x^2}\right) + 10x\cos\left(4\pi\sqrt{1-x^2}\right) + e^{4\pi x}x\right) + \\
&\quad 2e^{3\pi x}\left(2e^{\pi x}x - 3\sqrt{1-x^2}\sin\left(\pi\sqrt{1-x^2}\right)\right)\cos\left(4\pi\sqrt{1-x^2}\right)\Bigg)\Bigg) \Big/ \\
&\left(2\sqrt{1-x^2}\left(e^{3\pi x}\sin\left(\pi\sqrt{1-x^2}\right) + \sin\left(4\pi\sqrt{1-x^2}\right)\right)^2\right)
\end{aligned}$$

For x = 2:

[1/2+e^(-2Pi) cos(Pi*sqrt(1-4))+e^(-4Pi*2) cos(4Pi*sqrt(1-4))] / [e^(-2Pi) sin(Pi*sqrt(1-4)) +e^(-4Pi*2) sin(4Pi*sqrt(1-4))]

**Input**

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\sin\left(4\pi\sqrt{1-4}\right)}$$

**Exact result**

$$\frac{\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

**Decimal approximation**

$-3.1486345672307262772463771225186177046014974220521930734383968 0\ldots\, i$

-3.14863456723…i

**Polar coordinates**

$$r=\frac{e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}\ \text{(radius)},\quad \theta=-\frac{\pi}{2}\ \text{(angle)}$$

**Polar forms**

$$\frac{\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}$$

## Approximate form

$$\frac{e^{-(i\pi)/2}\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}$$

## Expanded form

$$\frac{1}{2\left(i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)\right)}+\frac{e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}+\frac{e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}$$

## Alternate forms

$$-\frac{i\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}$$

---

$$-\frac{i\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)\operatorname{csch}\left(\frac{\sqrt{3}\,\pi}{2}\right)\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)}{4\left(e^{6\pi}+2\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(3\sqrt{3}\,\pi\right)\right)}$$

---

$$-\frac{i\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)\operatorname{csch}\left(\sqrt{3}\,\pi\right)}{2\left(e^{6\pi}+e^{-3\sqrt{3}\,\pi}+e^{-\sqrt{3}\,\pi}+e^{\sqrt{3}\,\pi}+e^{3\sqrt{3}\,\pi}\right)}$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

$\operatorname{sech}(x)$ is the hyperbolic secant function

## Alternative representations

$$\frac{\frac{1}{2} + e^{-2\pi} \cos\left(\pi \sqrt{1-4}\right) + e^{-4\pi 2} \cos\left(4\pi \sqrt{1-4}\right)}{e^{-2\pi} \sin\left(\pi \sqrt{1-4}\right) + e^{-4\pi 2} \sin\left(4\pi \sqrt{1-4}\right)} = \frac{\frac{1}{2} + \cosh\left(4 i \pi \sqrt{-3}\right) e^{-8\pi} + \cosh\left(i \pi \sqrt{-3}\right) e^{-2\pi}}{\cos\left(\frac{\pi}{2} - 4\pi \sqrt{-3}\right) e^{-8\pi} + \cos\left(\frac{\pi}{2} - \pi \sqrt{-3}\right) e^{-2\pi}}$$

$$\frac{\frac{1}{2} + e^{-2\pi} \cos\left(\pi \sqrt{1-4}\right) + e^{-4\pi 2} \cos\left(4\pi \sqrt{1-4}\right)}{e^{-2\pi} \sin\left(\pi \sqrt{1-4}\right) + e^{-4\pi 2} \sin\left(4\pi \sqrt{1-4}\right)} = \frac{\frac{1}{2} + \cosh\left(-4 i \pi \sqrt{-3}\right) e^{-8\pi} + \cosh\left(-i \pi \sqrt{-3}\right) e^{-2\pi}}{\cos\left(\frac{\pi}{2} - 4\pi \sqrt{-3}\right) e^{-8\pi} + \cos\left(\frac{\pi}{2} - \pi \sqrt{-3}\right) e^{-2\pi}}$$

$$\frac{\frac{1}{2} + e^{-2\pi} \cos\left(\pi \sqrt{1-4}\right) + e^{-4\pi 2} \cos\left(4\pi \sqrt{1-4}\right)}{e^{-2\pi} \sin\left(\pi \sqrt{1-4}\right) + e^{-4\pi 2} \sin\left(4\pi \sqrt{1-4}\right)} = \frac{\frac{1}{2} + \cosh\left(-4 i \pi \sqrt{-3}\right) e^{-8\pi} + \cosh\left(-i \pi \sqrt{-3}\right) e^{-2\pi}}{-\cos\left(\frac{\pi}{2} + 4\pi \sqrt{-3}\right) e^{-8\pi} - \cos\left(\frac{\pi}{2} + \pi \sqrt{-3}\right) e^{-2\pi}}$$

## Series representations

$$\frac{\frac{1}{2} + e^{-2\pi} \cos\left(\pi \sqrt{1-4}\right) + e^{-4\pi 2} \cos\left(4\pi \sqrt{1-4}\right)}{e^{-2\pi} \sin\left(\pi \sqrt{1-4}\right) + e^{-4\pi 2} \sin\left(4\pi \sqrt{1-4}\right)} = -\frac{i \left(e^{8\pi} + \sum_{k=0}^{\infty} \frac{2 \times 3^k \left(16^k + e^{6\pi}\right) \pi^{2k}}{(2k)!}\right)}{2 \sum_{k=0}^{\infty} \frac{3^{1/2+k} \left(4^{1+2k} + e^{6\pi}\right) \pi^{1+2k}}{(1+2k)!}}$$

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)}=$$

$$\frac{1+2\sum_{j=0}^{\infty}e^{-8\pi}\sqrt{\pi}\left(\operatorname{Res}_{s=-j}\frac{(-12)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}+e^{6\pi}\left(\operatorname{Res}_{s=-j}\frac{\left(-\frac{3}{4}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)\right)}{2\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}}$$

---

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)}=$$

$$-\frac{i\left(e^{8\pi}+\sum_{k=0}^{\infty}\frac{2\times 3^{k}\left(16^{k}+e^{6\pi}\right)\pi^{2k}}{(2k)!}\right)}{\sqrt{3}\,\pi^{3/2}\sum_{j=0}^{\infty}\left(4\left(\operatorname{Res}_{s=-j}\frac{(-12)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)+e^{6\pi}\left(\operatorname{Res}_{s=-j}\frac{\left(-\frac{3}{4}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\right)}$$

## Integral representations

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)}=$$

$$-\frac{i\left(2+2e^{6\pi}+e^{8\pi}+\int_0^1 2\sqrt{3}\,\pi\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi t\right)+4\sinh\left(4\sqrt{3}\,\pi t\right)\right)dt\right)}{2\sqrt{3}\,\pi\int_0^1\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi t\right)+4\cosh\left(4\sqrt{3}\,\pi t\right)\right)dt}$$

---

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)}=$$

$$-\frac{i\left(e^{8\pi}+2e^{6\pi}\int_{\frac{i\pi}{2}}^{\sqrt{3}\,\pi}\sinh(t)\,dt+2\int_{\frac{i\pi}{2}}^{4\sqrt{3}\,\pi}\sinh(t)\,dt\right)}{2\sqrt{3}\,\pi\int_0^1\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi t\right)+4\cosh\left(4\sqrt{3}\,\pi t\right)\right)dt}$$

---

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)}=\frac{1+2\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+e^{(45\pi^2)/(4s)}\right)}{2\sqrt{\pi}\sqrt{s}}\,ds}{2\int_0^1 i\sqrt{3}\,e^{-8\pi}\,\pi\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi\,t\right)+4\cosh\left(4\sqrt{3}\,\pi\,t\right)\right)dt}\quad\text{for }\gamma>0$$

## Multiple-argument formulas

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)}=-\frac{i\left(e^{8\pi}+2e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}$$

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)}=-\frac{i\left(e^{8\pi}+2e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)\operatorname{csch}\left(\sqrt{3}\,\pi\right)}{2\left(e^{6\pi}+2\left(\cosh\left(\sqrt{3}\,\pi\right)+\cosh\left(3\sqrt{3}\,\pi\right)\right)\right)}$$

$$\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)}=\frac{\frac{1}{2}+e^{-2\pi}\left(-1+2\cosh^2\left(\frac{\sqrt{3}\,\pi}{2}\right)\right)+e^{-8\pi}\left(-1+2\cosh^2\left(2\sqrt{3}\,\pi\right)\right)}{2\,i\,e^{-2\pi}\cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)\sinh\left(\frac{\sqrt{3}\,\pi}{2}\right)+2\,i\,e^{-8\pi}\cosh\left(2\sqrt{3}\,\pi\right)\sinh\left(2\sqrt{3}\,\pi\right)}$$

We obtain from:

(sqrt2+sqrt(1+2)) / (sqrt(1-2))

## Input

$$\frac{\sqrt{2}+\sqrt{1+2}}{\sqrt{1-2}}$$

## Result

$-(\sqrt{2}+\sqrt{3})\,i$

## Decimal approximation

$-3.1462643699419723423291350657155704455124771291873287012324867 1\ldots\,i$

-3.1462643699….i

## Polar coordinates

$r = \sqrt{2}+\sqrt{3}$ (radius), $\theta = -\frac{\pi}{2}$ (angle)

## Polar forms

$$(\sqrt{2}+\sqrt{3})\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right)$$

---

Approximate form

$(\sqrt{2}+\sqrt{3})\,e^{-(i\pi)/2}$

## Alternate forms

$-i\sqrt{5+2\sqrt{6}}$

---

$i(-\sqrt{2}-\sqrt{3})$

## Minimal polynomial

$x^4+10x^2+1$

## Expanded form

$-i\sqrt{2}-i\sqrt{3}$

Multiplying the two expressions, and adding

$$\frac{1}{3}(\log(8\pi)-\pi)\approx 0.02752625$$

we obtain:

(((-(([1/2+e^(-2Pi) cos(Pi*sqrt(1-4))+e^(-4Pi*2) cos(4Pi*sqrt(1-4))] / [e^(-2Pi) sin(Pi*sqrt(1-4)) +e^(-4Pi*2) sin(4Pi*sqrt(1-4))] [(sqrt2+sqrt(1+2)) / (sqrt(1-2))]-(1/3 (log(8 π) - π)))))))

**Input**

$$-\left(\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\sin\left(4\pi\sqrt{1-4}\right)}\times\frac{\sqrt{2}+\sqrt{1+2}}{\sqrt{1-2}}-\frac{1}{3}(\log(8\pi)-\pi)\right)$$

log(x) is the natural logarithm

**Exact result**

$$\frac{1}{3}(\log(8\pi)-\pi)+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

**Decimal approximation**

9.9339630108255100545025949500049492394145213253860818276546254259
…

9.933963…. result very near to the entropy black hole ln(20619) = 9.933968….

## Alternate forms

$$\frac{1}{3}\left(-\pi+\log(8\pi)+\frac{3\left(\sqrt{2}+\sqrt{3}\right)\left(e^{8\pi}+2e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}\right)$$

---

$$\frac{1}{3}(-\pi+\log(8)+\log(\pi))+\frac{\left(\sqrt{2}+\sqrt{3}\right)e^{8\pi}\left(\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)\right)}{e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)}$$

---

$$-\frac{\pi}{3}+\log(2)+\frac{\log(\pi)}{3}-\frac{\left(-\sqrt{2}-\sqrt{3}\right)\left(e^{8\pi}+2e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)\operatorname{csch}\left(\sqrt{3}\,\pi\right)}{2\left(e^{6\pi}+e^{-3\sqrt{3}\,\pi}+e^{-\sqrt{3}\,\pi}+e^{\sqrt{3}\,\pi}+e^{3\sqrt{3}\,\pi}\right)}$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

## Expanded form

$$-\frac{\pi}{3}+\frac{1}{3}\log(8\pi)+\frac{i}{\sqrt{2}\left(i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)\right)}+\frac{i\sqrt{3}}{2\left(i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)\right)}+\frac{i\sqrt{2}\,e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}+\frac{i\sqrt{3}\,e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}+\frac{i\sqrt{2}\,e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}+\frac{i\sqrt{3}\,e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}$$

## Alternative representations

$$-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)=\frac{1}{3}(-\pi+\log(a)\log_a(8\pi))-\frac{\left(\frac{1}{2}+\cosh\left(-4i\pi\sqrt{-3}\right)e^{-8\pi}+\cosh\left(-i\pi\sqrt{-3}\right)e^{-2\pi}\right)\left(\sqrt{2}+\sqrt{3}\right)}{\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}}$$

---

$$-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)=\frac{1}{3}\left(-\pi+\log_e(8\pi)\right)-\left(\left(\frac{1}{2}+\frac{1}{2}e^{-2\pi}\left(e^{-i\left(\pi\sqrt{-3}\right)}+e^{i\pi\sqrt{-3}}\right)+\frac{1}{2}e^{-8\pi}\left(e^{-4i\pi\sqrt{-3}}+e^{4i\pi\sqrt{-3}}\right)\right)\left(\sqrt{2}+\sqrt{3}\right)\right)\Big/\left(\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}\right)$$

---

$$-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)=\frac{1}{3}(-\pi+\log(a)\log_a(8\pi))-\left(\left(\frac{1}{2}+\frac{1}{2}e^{-2\pi}\left(e^{-i\left(\pi\sqrt{-3}\right)}+e^{i\pi\sqrt{-3}}\right)+\frac{1}{2}e^{-8\pi}\left(e^{-4i\pi\sqrt{-3}}+e^{4i\pi\sqrt{-3}}\right)\right)\left(\sqrt{2}+\sqrt{3}\right)\right)\Big/\left(\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}\right)$$

## Series representations

$$-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}\left(\log(8\pi)-\pi\right)\right)=$$

$$-\left(\left(-3\,i\sqrt{2}-3\,i\sqrt{3}-6\,i\sqrt{2}\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}-6\,i\sqrt{3}\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}+2\pi\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}-2\log(-1+8\pi)\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}+2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{\left(-\frac{1}{-1+8\pi}\right)^{k_1}\left(\frac{i\,3^{1/2(1+2k_2)}e^{-2\pi}\pi^{1+2k_2}}{(1+2k_2)!}+\frac{i\,3^{1/2(1+2k_2)}e^{-8\pi}(4\pi)^{1+2k_2}}{(1+2k_2)!}\right)}{k_1}\right)\Bigg/\left(6\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}\right)\right)$$

---

$$-\Bigg(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-$$

$$\frac{1}{3}(\log(8\pi)-\pi)\Bigg)=-\Bigg(\Bigg(\Bigg(-3i\sqrt{2}-3i\sqrt{3}-$$

$$6i\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{i\,e^{-2\pi}\left(-\frac{i\pi}{2}+\sqrt{3}\,\pi\right)^{1+2k}}{(1+2k)!}+\frac{i\,e^{-8\pi}\left(-\frac{i\pi}{2}+4\sqrt{3}\,\pi\right)^{1+2k}}{(1+2k)!}\right)-$$

$$6i\sqrt{3}\sum_{k=0}^{\infty}\left(\frac{i\,e^{-2\pi}\left(-\frac{i\pi}{2}+\sqrt{3}\,\pi\right)^{1+2k}}{(1+2k)!}+\frac{i\,e^{-8\pi}\left(-\frac{i\pi}{2}+4\sqrt{3}\,\pi\right)^{1+2k}}{(1+2k)!}\right)+$$

$$2\pi\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}\,e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}-2\log(-1+8\pi)$$

$$\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}\,e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}+2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}$$

$$\frac{\left(-\frac{1}{-1+8\pi}\right)^{k_1}\left(\frac{i\,3^{1/2(1+2k_2)}\,e^{-2\pi}\,\pi^{1+2k_2}}{(1+2k_2)!}+\frac{i\,3^{1/2(1+2k_2)}\,e^{-8\pi}\,(4\pi)^{1+2k_2}}{(1+2k_2)!}\right)}{k_1}\Bigg)$$

$$\Bigg/\left(6\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}\,e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}\right)\Bigg)$$

---

$$
-\Bigg(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\Bigg)=
$$

$$
-\Bigg(\Bigg(\Bigg(-3i\sqrt{2}-3i\sqrt{3}-6i\sqrt{2}\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}-
6i\sqrt{3}\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}+
2\pi\sum_{k=0}^{\infty}-\frac{e^{-8\pi}\left(e^{6\pi}\left(\left(-\frac{i}{2}+\sqrt{3}\right)\pi\right)^{2k}+\left(\frac{1}{2}\left(-i+8\sqrt{3}\right)\pi\right)^{2k}\right)}{(2k)!}-2\log(
-1+8\pi)\sum_{k=0}^{\infty}-\frac{e^{-8\pi}\left(e^{6\pi}\left(\left(-\frac{i}{2}+\sqrt{3}\right)\pi\right)^{2k}+\left(\frac{1}{2}\left(-i+8\sqrt{3}\right)\pi\right)^{2k}\right)}{(2k)!}+
2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{\left(-\frac{1}{-1+8\pi}\right)^{k_1}\left(-\frac{e^{-2\pi}\left(-\frac{i\pi}{2}+\sqrt{3}\,\pi\right)^{2k_2}}{(2k_2)!}-\frac{e^{-8\pi}\left(-\frac{i\pi}{2}+4\sqrt{3}\,\pi\right)^{2k_2}}{(2k_2)!}\right)}{k_1}\Bigg)\Bigg/
\left(6\sum_{k=0}^{\infty}-\frac{e^{-8\pi}\left(e^{6\pi}\left(\left(-\frac{i}{2}+\sqrt{3}\right)\pi\right)^{2k}+\left(\frac{1}{2}\left(-i+8\sqrt{3}\right)\pi\right)^{2k}\right)}{(2k)!}\right)\Bigg)
$$

Indeed:

exp((((-(([1/2+e^(-2Pi) cos(Pi*sqrt(1-4))+e^(-4Pi*2) cos(4Pi*sqrt(1-4))] / [e^(-2Pi) sin(Pi*sqrt(1-4)) +e^(-4Pi*2) sin(4Pi*sqrt(1-4))] [(sqrt2+sqrt(1+2))/(sqrt(1-2))]-(1/3(ln(8π)-π))))))))

**Input**

$$
\exp\left(-\left(\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\sin\left(4\pi\sqrt{1-4}\right)}\times\frac{\sqrt{2}+\sqrt{1+2}}{\sqrt{1-2}}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)
$$

log(x) is the natural logarithm

## Exact result

$$e^{\frac{1}{3}(\log(8\pi)-\pi)+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}}$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

## Decimal approximation

20618.891771891103188455356507399204107005118695090696192691013920

…

**20618**.8917718911….

## Alternate forms

$$2\sqrt[3]{\pi}\,\exp\left(\frac{\left(\sqrt{2}+\sqrt{3}\right)\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}-\frac{\pi}{3}\right)$$

---

$$e^{\frac{1}{3}(-\pi+\log(8)+\log(\pi))+\frac{\left(\sqrt{2}+\sqrt{3}\right)e^{8\pi}\left(\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)\right)}{e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)}}$$

---

$$2\sqrt[3]{\pi}\,\exp\left(-\frac{\pi}{3}+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}\right)$$

## Alternative representations

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)=\exp\left(\frac{1}{3}(-\pi+\log(a)\log_a(8\pi))-\frac{\left(\frac{1}{2}+\cosh\left(-4i\pi\sqrt{-3}\right)e^{-8\pi}+\cosh\left(-i\pi\sqrt{-3}\right)e^{-2\pi}\right)\left(\sqrt{2}+\sqrt{3}\right)}{\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}}\right)$$

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)=\exp\left(\frac{1}{3}\left(-\pi+\log_e(8\pi)\right)-\left(\left(\frac{1}{2}+\frac{1}{2}e^{-2\pi}\left(e^{-i\left(\pi\sqrt{-3}\right)}+e^{i\pi\sqrt{-3}}\right)+\frac{1}{2}e^{-8\pi}\left(e^{-4i\pi\sqrt{-3}}+e^{4i\pi\sqrt{-3}}\right)\right)\left(\sqrt{2}+\sqrt{3}\right)\right)\Big/\left(\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}\right)\right)$$

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)=\exp\left(\frac{1}{3}(-\pi+\log(a)\log_a(8\pi))-\left(\left(\frac{1}{2}+\frac{1}{2}e^{-2\pi}\left(e^{-i\left(\pi\sqrt{-3}\right)}+e^{i\pi\sqrt{-3}}\right)+\frac{1}{2}e^{-8\pi}\left(e^{-4i\pi\sqrt{-3}}+e^{4i\pi\sqrt{-3}}\right)\right)\left(\sqrt{2}+\sqrt{3}\right)\right)\Big/\left(\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}\right)\right)$$

## Series representations

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)=$$

$$\exp\left(-\frac{\pi}{3}-\frac{1}{3}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{1-8\pi}\right)^k}{k}+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}\right)}{\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}}\right)\sqrt[3]{-1+8\pi}$$

---

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)=$$

$$\exp\left(-\frac{\pi}{3}-\frac{1}{3}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{1-8\pi}\right)^k}{k}+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}\right)}{\sum_{k=0}^{\infty}-\frac{e^{-8\pi}\left(e^{6\pi}\left(\left(-\frac{i}{2}+\sqrt{3}\right)\pi\right)^{2k}+\left(\frac{1}{2}\left(-i+8\sqrt{3}\right)\pi\right)^{2k}\right)}{(2k)!}}\right)\sqrt[3]{-1+8\pi}$$

---

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)=\exp\left(-\frac{\pi}{3}-\frac{1}{3}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{1-8\pi}\right)^k}{k}+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\sum_{k=0}^{\infty}\left(\frac{i\,e^{-2\pi}\left(-\frac{i\pi}{2}+\sqrt{3}\,\pi\right)^{1+2k}}{(1+2k)!}+\frac{i\,e^{-8\pi}\left(-\frac{i\pi}{2}+4\sqrt{3}\,\pi\right)^{1+2k}}{(1+2k)!}\right)\right)}{\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}}\right)\sqrt[3]{-1+8\pi}$$

## Integral representations

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)=$$

$$2\exp\left(-\frac{\pi}{3}+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+e^{(45\pi^2)/(4s)}\right)}{2\sqrt{\pi}\sqrt{s}}\,ds\right)}{\int_0^1 i\sqrt{3}\,e^{-8\pi}\pi\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi t\right)+4\cosh\left(4\sqrt{3}\,\pi t\right)\right)dt}\right)\sqrt[3]{\pi}\quad\text{for }\gamma>0$$

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}(\log(8\pi)-\pi)\right)\right)=$$

$$2\exp\left(-\frac{\pi}{3}+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+e^{(45\pi^2)/(4s)}\right)}{2\sqrt{\pi}\sqrt{s}}\,ds\right)}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+4e^{(45\pi^2)/(4s)}\right)\sqrt{3\pi}}{4s^{3/2}}\,ds}\right)\sqrt[3]{\pi}\quad\text{for }\gamma>0$$

$$\exp\left(-\left(\frac{\left(\sqrt{2}+\sqrt{1+2}\right)\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)}{\sqrt{1-2}\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)}-\frac{1}{3}\left(\log(8\pi)-\pi\right)\right)\right)=\exp\left(-\frac{\pi}{3}+\frac{1}{3}\int_1^{8\pi}\frac{1}{t}\,dt+\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+e^{(45\pi^2)/(4s)}\right)}{2\sqrt{\pi}\sqrt{s}}\,ds\right)}{\int_0^1 i\sqrt{3}\,e^{-8\pi}\,\pi\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi t\right)+4\cosh\left(4\sqrt{3}\,\pi t\right)\right)dt}\right)\quad\text{for } \gamma>0$$

Or:

(((-((([1/2+e^(-2Pi) cos(Pi*sqrt(1-4))+e^(-4Pi*2) cos(4Pi*sqrt(1-4))] / [e^(-2Pi) sin(Pi*sqrt(1-4)) +e^(-4Pi*2) sin(4Pi*sqrt(1-4))] [(sqrt2+sqrt(1+2)) / (sqrt(1-2))])))))

## Input

$$-\left(\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\sin\left(4\pi\sqrt{1-4}\right)}\times\frac{\sqrt{2}+\sqrt{1+2}}{\sqrt{1-2}}\right)$$

## Exact result

$$\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

## Decimal approximation

9.9064367528456957665251015058555873955223127198457579900210525900
…

9.9064367528…. result quite near to the entropy black hole ln(20619) = 9.933968….

## Alternate forms

$$\frac{\left(\sqrt{2}+\sqrt{3}\right)\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}$$

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$$\frac{\left(\sqrt{2}+\sqrt{3}\right)\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)\operatorname{csch}\left(\frac{\sqrt{3}\,\pi}{2}\right)\operatorname{sech}\left(\frac{\sqrt{3}\,\pi}{2}\right)}{4\left(e^{6\pi}+2\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(3\sqrt{3}\,\pi\right)\right)}$$

---

$$-\frac{\left(-\sqrt{2}-\sqrt{3}\right)\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)\operatorname{csch}\left(\sqrt{3}\,\pi\right)}{2\left(e^{6\pi}+e^{-3\sqrt{3}\,\pi}+e^{-\sqrt{3}\,\pi}+e^{\sqrt{3}\,\pi}+e^{3\sqrt{3}\,\pi}\right)}$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

$\operatorname{sech}(x)$ is the hyperbolic secant function

## Expanded form

$$\frac{i}{\sqrt{2}\left(i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)\right)}+\frac{i\sqrt{3}}{2\left(i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)\right)}+\frac{i\sqrt{2}\,e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}+\frac{i\sqrt{3}\,e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}+\frac{i\sqrt{2}\,e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}+\frac{i\sqrt{3}\,e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}$$

## Alternative representations

$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}=$$
$$-\frac{\left(\frac{1}{2}+\cosh\left(4\,i\,\pi\sqrt{-3}\right)e^{-8\pi}+\cosh\left(i\,\pi\sqrt{-3}\right)e^{-2\pi}\right)\left(\sqrt{2}+\sqrt{3}\right)}{\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}}$$

$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}=$$
$$-\frac{\left(\frac{1}{2}+\cosh\left(-4\,i\,\pi\sqrt{-3}\right)e^{-8\pi}+\cosh\left(-i\,\pi\sqrt{-3}\right)e^{-2\pi}\right)\left(\sqrt{2}+\sqrt{3}\right)}{\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}}$$

$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}=$$
$$-\frac{\left(\frac{1}{2}+\cosh\left(-4\,i\,\pi\sqrt{-3}\right)e^{-8\pi}+\cosh\left(-i\,\pi\sqrt{-3}\right)e^{-2\pi}\right)\left(\sqrt{2}+\sqrt{3}\right)}{\left(-\cos\left(\frac{\pi}{2}+4\pi\sqrt{-3}\right)e^{-8\pi}-\cos\left(\frac{\pi}{2}+\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}}$$

## Series representations

$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}=$$
$$\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(1+2\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}\right)}{2\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}}$$

$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}=$$

$$\left(i\left(\sqrt{2}+\sqrt{3}\right)\left(1+2\sum_{j=0}^{\infty}e^{-8\pi}\sqrt{\pi}\left(\operatorname{Res}_{s=-j}\frac{(-12)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}+e^{6\pi}\left(\operatorname{Res}_{s=-j}\frac{\left(-\frac{3}{4}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)\right)\right)\right)\Bigg/\left(2\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}\right)$$

$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}=$$

$$\frac{\left(3+\sqrt{6}\right)\left(e^{8\pi}+\sum_{k=0}^{\infty}\frac{2\times 3^{k}\left(16^{k}+e^{6\pi}\right)\pi^{2k}}{(2k)!}\right)}{3\pi^{3/2}\sum_{j=0}^{\infty}\left(4\left(\operatorname{Res}_{s=-j}\frac{(-12)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)+e^{6\pi}\left(\operatorname{Res}_{s=-j}\frac{\left(-\frac{3}{4}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\right)}$$

## Integral representations

$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}=$$

$$\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(1+2\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+e^{(45\pi^2)/(4s)}\right)}{2\sqrt{\pi}\sqrt{s}}\,ds\right)}{2\int_0^1 i\sqrt{3}\,e^{-8\pi}\pi\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi t\right)+4\cosh\left(4\sqrt{3}\,\pi t\right)\right)dt}\quad\text{for }\gamma>0$$

$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}=$$

$$\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(1+2\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+e^{(45\pi^2)/(4s)}\right)}{2\sqrt{\pi}\sqrt{s}}\,ds\right)}{2\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+4e^{(45\pi^2)/(4s)}\right)\sqrt{3\pi}}{4s^{3/2}}\,ds}\quad\text{for }\gamma>0$$

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$$-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}} =$$

$$\left(i\left(\sqrt{2}+\sqrt{3}\right)e^{-8\pi}\left(2+2e^{6\pi}+e^{8\pi}+ 2\sqrt{3}\,e^{6\pi}\pi\int_0^1\sinh\left(\sqrt{3}\,\pi t\right)dt+8\sqrt{3}\,\pi\int_0^1\sinh\left(4\sqrt{3}\,\pi t\right)dt\right)\right)\Big/ \left(2\int_0^1 i\sqrt{3}\,e^{-8\pi}\pi\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi t\right)+4\cosh\left(4\sqrt{3}\,\pi t\right)\right)dt\right)$$

From which, after some calculations:

exp(((-(([1/2+e^(-2Pi) cos(Pi*sqrt(1-4))+e^(-4Pi*2) cos(4Pi*sqrt(1-4))] / [e^(-2Pi) sin(Pi*sqrt(1-4)) +e^(-4Pi*2) sin(4Pi*sqrt(1-4))] [(sqrt2+sqrt(1+2)) / (sqrt(1-2))])))))+24^2-16

**Input**

$$\exp\left(-\left(\frac{\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\cos\left(4\pi\sqrt{1-4}\right)}{e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi\times 2}\sin\left(4\pi\sqrt{1-4}\right)}\times\frac{\sqrt{2}+\sqrt{1+2}}{\sqrt{1-2}}\right)\right)+24^2-16$$

**Exact result**

$$560+\exp\left(\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)\right)}{i\,e^{-2\pi}\sinh\left(\sqrt{3}\,\pi\right)+i\,e^{-8\pi}\sinh\left(4\sqrt{3}\,\pi\right)}\right)$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

**Decimal approximation**

20619.071069584103154622925889275972985831846654464449096287777516
…

**20619**.0710695841….

## Alternate forms

$$\exp\left(\frac{\left(\sqrt{2}+\sqrt{3}\right)\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)}{2\left(e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)\right)}\right)+560$$

$$\exp\left(\frac{\left(\sqrt{2}+\sqrt{3}\right)e^{8\pi}\left(\frac{1}{2}+e^{-2\pi}\cosh\left(\sqrt{3}\,\pi\right)+e^{-8\pi}\cosh\left(4\sqrt{3}\,\pi\right)\right)}{e^{6\pi}\sinh\left(\sqrt{3}\,\pi\right)+\sinh\left(4\sqrt{3}\,\pi\right)}\right)+560$$

$$\exp\left(-\frac{\left(-\sqrt{2}-\sqrt{3}\right)\left(e^{8\pi}+2\,e^{6\pi}\cosh\left(\sqrt{3}\,\pi\right)+2\cosh\left(4\sqrt{3}\,\pi\right)\right)\operatorname{csch}\left(\sqrt{3}\,\pi\right)}{2\left(e^{6\pi}+e^{-3\sqrt{3}\,\pi}+e^{-\sqrt{3}\,\pi}+e^{\sqrt{3}\,\pi}+e^{3\sqrt{3}\,\pi}\right)}\right)+560$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

## Alternative representations

$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+24^2-16=-16+\exp\left(-\left(\left(\left(\frac{1}{2}+\frac{1}{2}e^{-2\pi}\left(e^{-i\left(\pi\sqrt{-3}\right)}+e^{i\pi\sqrt{-3}}\right)+\frac{1}{2}e^{-8\pi}\left(e^{-4i\pi\sqrt{-3}}+e^{4i\pi\sqrt{-3}}\right)\right)\left(\sqrt{2}+\sqrt{3}\right)\right)\Big/\left(\left(\cos\left(\frac{\pi}{2}-4\pi\sqrt{-3}\right)e^{-8\pi}+\cos\left(\frac{\pi}{2}-\pi\sqrt{-3}\right)e^{-2\pi}\right)\sqrt{-1}\right)\right)\right)+24^2$$

$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+24^2-16=\exp\left(-\left(\left(\left(\frac{1}{2}+\frac{1}{2}z^{-2\pi}\left(e^{i\left(\pi\sqrt{1-4}\right)}+e^{-i\left(\pi\sqrt{1-4}\right)}\right)+\frac{1}{2}z^{-4\pi 2}\left(e^{i\left(4\pi\sqrt{1-4}\right)}+e^{-i\left(4\pi\sqrt{1-4}\right)}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)\right)\Big/\left(\left(z^{-2\pi}\left(-\cos\left(\frac{\pi}{2}+\pi\sqrt{1-4}\right)\right)+z^{-4\pi 2}\left(-\cos\left(\frac{\pi}{2}+4\pi\sqrt{1-4}\right)\right)\right)\sqrt{1-2}\right)\right)\right)+24^2-16 \quad \text{for } z=e$$

---

$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+24^2-16=\exp\left(-\left(\left(\left(\frac{1}{2}+\frac{1}{2}z^{-2\pi}\left(e^{i\left(\pi\sqrt{1-4}\right)}+e^{-i\left(\pi\sqrt{1-4}\right)}\right)+\frac{1}{2}z^{-4\pi 2}\left(e^{i\left(4\pi\sqrt{1-4}\right)}+e^{-i\left(4\pi\sqrt{1-4}\right)}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)\right)\Big/\left(\left(z^{-2\pi}\cos\left(\frac{\pi}{2}-\pi\sqrt{1-4}\right)+z^{-4\pi 2}\cos\left(\frac{\pi}{2}-4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}\right)\right)\right)+24^2-16 \quad \text{for } z=e$$

## Series representations

$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+24^2-16=560+\exp\left(\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}\right)}{\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}}\right)$$

---

$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+24^2-16=560+\exp\left(\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\sum_{k=0}^{\infty}\left(\frac{i\,e^{-2\pi}\left(-\frac{i\pi}{2}+\sqrt{3}\,\pi\right)^{1+2k}}{(1+2k)!}+\frac{i\,e^{-8\pi}\left(-\frac{i\pi}{2}+4\sqrt{3}\,\pi\right)^{1+2k}}{(1+2k)!}\right)\right)}{\sum_{k=0}^{\infty}\frac{i\,3^{1/2+k}e^{-8\pi}\left(4^{1+2k}+e^{6\pi}\right)\pi^{1+2k}}{(1+2k)!}}\right)$$

---

$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+ 24^2-16=560+\exp\left(\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\sum_{k=0}^{\infty}\frac{3^k e^{-8\pi}\left(16^k+e^{6\pi}\right)\pi^{2k}}{(2k)!}\right)}{\sum_{k=0}^{\infty}\left(-\frac{e^{-2\pi}\left(-\frac{i\pi}{2}+\sqrt{3}\,\pi\right)^{2k}}{(2k)!}-\frac{e^{-8\pi}\left(-\frac{i\pi}{2}+4\sqrt{3}\,\pi\right)^{2k}}{(2k)!}\right)}\right)$$

## Integral representations

$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+ 24^2-16= 560+\exp\left(\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+e^{(45\pi^2)/(4s)}\right)}{2\sqrt{\pi}\,\sqrt{s}}\,ds\right)}{\int_0^1 i\sqrt{3}\,e^{-8\pi}\,\pi\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi t\right)+4\cosh\left(4\sqrt{3}\,\pi t\right)\right)dt}\right)$$

for $\gamma > 0$

$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+ 24^2-16= 560+\exp\left(\left(i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+e^{-2\pi}\left(1+\sqrt{3}\,\pi\int_0^1\sinh\left(\sqrt{3}\,\pi t\right)dt\right)+e^{-8\pi}\left(1+4\sqrt{3}\,\pi\int_0^1\sinh\left(4\sqrt{3}\,\pi t\right)dt\right)\right)\right)\Big/\left(\int_0^1 i\sqrt{3}\,e^{-8\pi}\,\pi\left(e^{6\pi}\cosh\left(\sqrt{3}\,\pi t\right)+4\cosh\left(4\sqrt{3}\,\pi t\right)\right)dt\right)\right)$$

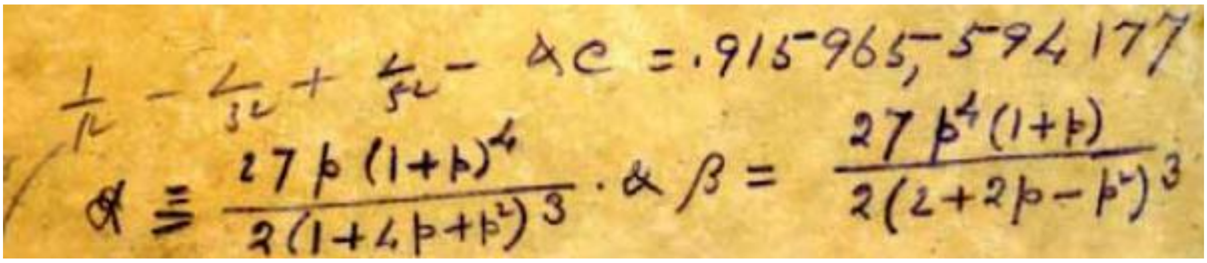
$$\exp\left(-\frac{\left(\frac{1}{2}+e^{-2\pi}\cos\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\cos\left(4\pi\sqrt{1-4}\right)\right)\left(\sqrt{2}+\sqrt{1+2}\right)}{\left(e^{-2\pi}\sin\left(\pi\sqrt{1-4}\right)+e^{-4\pi 2}\sin\left(4\pi\sqrt{1-4}\right)\right)\sqrt{1-2}}\right)+$$

$$24^2-16=$$

$$560+\exp\left(\frac{i\left(\sqrt{2}+\sqrt{3}\right)\left(\frac{1}{2}+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+e^{(45\pi^2)/(4s)}\right)}{2\sqrt{\pi}\sqrt{s}}\,ds\right)}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-8\pi+(3\pi^2)/(4s)+s}\left(e^{6\pi}+4e^{(45\pi^2)/(4s)}\right)\sqrt{3\pi}}{4s^{3/2}}\,ds}\right)$$

for $\gamma > 0$

From:



$\frac{1}{1^2}-\frac{1}{3^2}+\frac{1}{5^2}-$ &c $=.915965594177$

$\alpha = \frac{27p(1+p)^4}{2(1+4p+p^2)^3}$ & $\beta = \frac{27p^4(1+p)}{2(2+2p-p^2)^3}$

(See: *Analyzing further equations concerning the Ramanujan Manuscript Book 1. New possible mathematical connections with various sectors of Number Theory and Supersymmetry Breaking. VI – M.Nardelli and A.Nardelli –*

https://www.academia.edu/62643635/ )

For p = 11.8

$$\frac{27\times 11.8\,(11.8+1)^4}{2\,(11.8\,(11.8+4)+1)^3}$$

= 0.649335022555….

For p = 3.3 :

$$\frac{-27\times 3.3^4(3.3+1)}{2\left(3.3^2-2\times 3.3-2\right)^3}$$

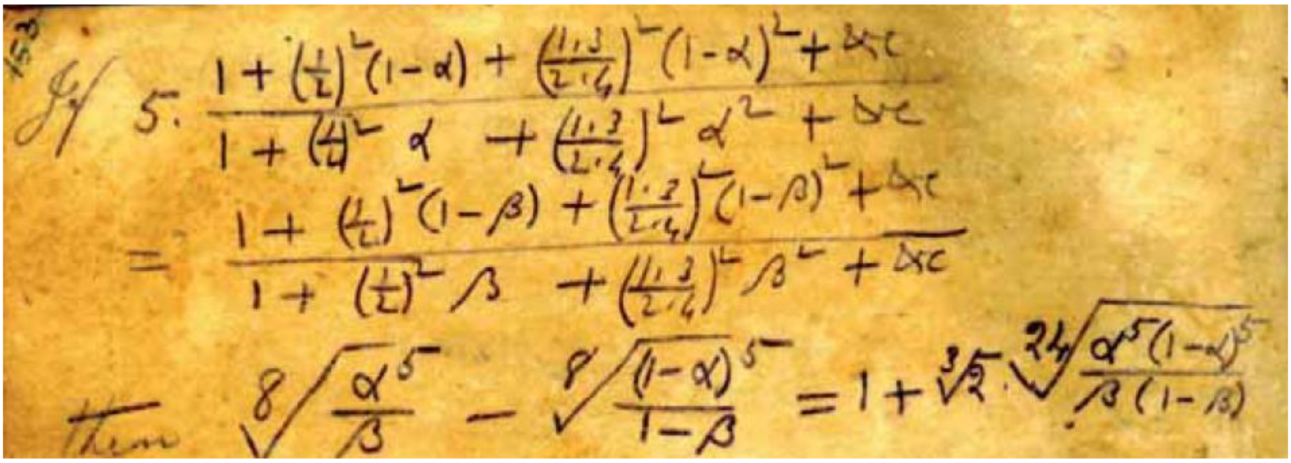
= -573.2598643399….

For:

0.649335…= α

-573.259864339 = β

From:

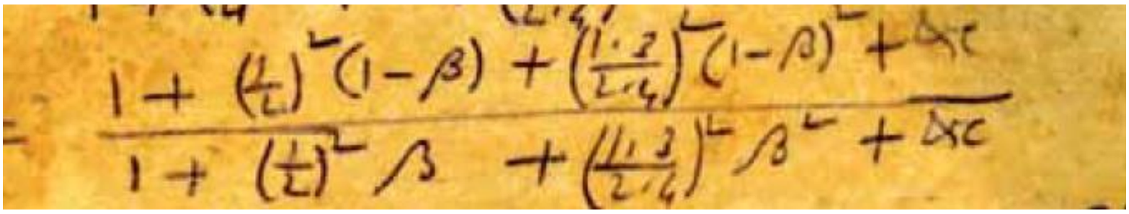


If 5:

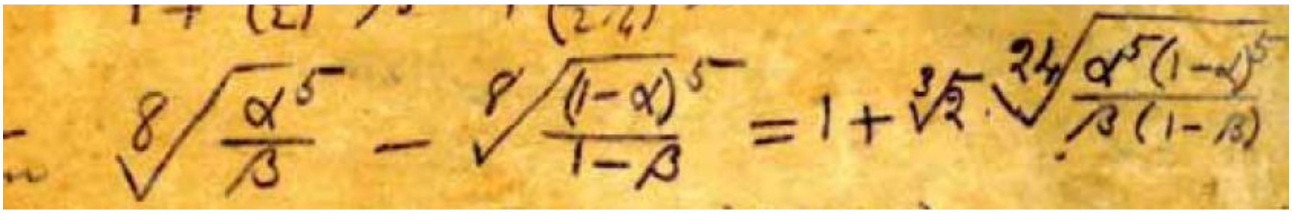
$$\frac{1+\left(\frac{1}{2}\right)^2(1-\alpha)+\left(\frac{1.3}{2.4}\right)^2(1-\alpha)^2+\&c}{1+\left(\frac{1}{2}\right)^2\alpha+\left(\frac{1.3}{2.4}\right)^2\alpha^2+\&c}=\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{1.2}{2.4}\right)^2(1-\beta)^2+\&c}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{1.3}{2.4}\right)^2\beta^2+\&c}$$

then

$$\sqrt[8]{\frac{\alpha^5}{\beta}}-\sqrt[8]{\frac{(1-\alpha)^5}{1-\beta}}=1+\sqrt[3]{2}\,\sqrt[24]{\frac{\alpha^5(1-\alpha)^5}{\beta(1-\beta)}}$$



$$=\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{1.2}{2.4}\right)^2(1-\beta)^2+\&c}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{1.3}{2.4}\right)^2\beta^2+\&c}$$



$$\sqrt[8]{\frac{\alpha^5}{\beta}} - \sqrt[8]{\frac{(1-\alpha)^5}{1-\beta}} = 1 + \sqrt[3]{2}\cdot\sqrt[24]{\frac{\alpha^5(1-\alpha)^5}{\beta(1-\beta)}}$$

Pag.273 Manuscript 1

We have:

(α^5/β)^(1/8) – ((1-α)^5 / (1-β))^(1/8) = 1+2^(1/3)*(((α^5(1-α)^5))/(β(1-β)))^(1/24)

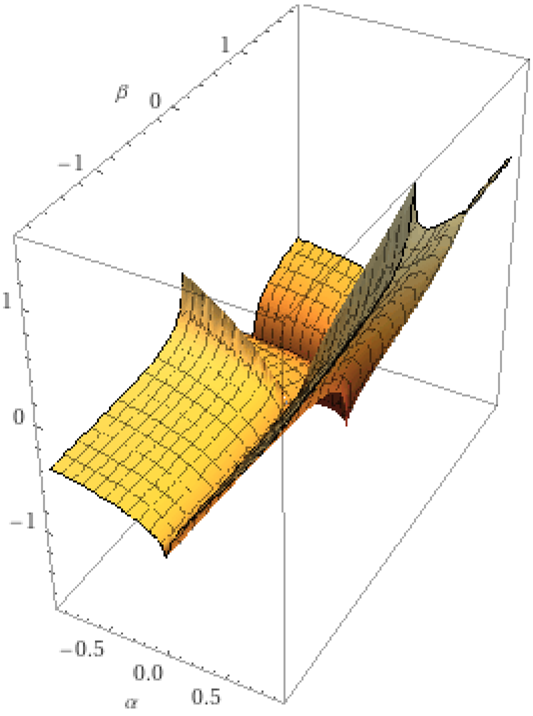
(α^5/β)^(1/8) – ((1-α)^5 / (1-β))^(1/8)

**Input**

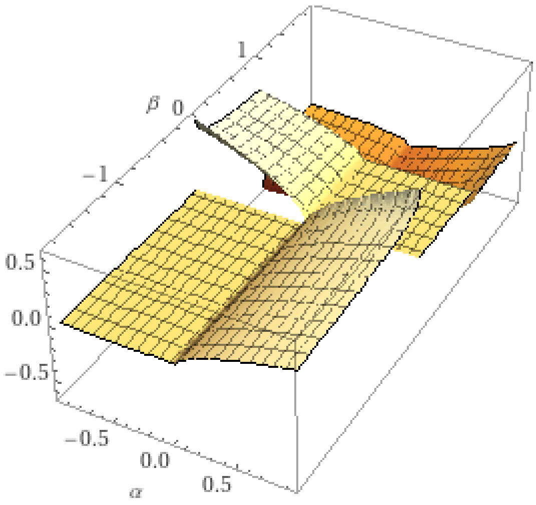
$$\sqrt[8]{\frac{\alpha^5}{\beta}} - \sqrt[8]{\frac{(1-\alpha)^5}{1-\beta}}$$

**3D plots**

**Real part** (figures that can be related to the D-branes/Instantons)

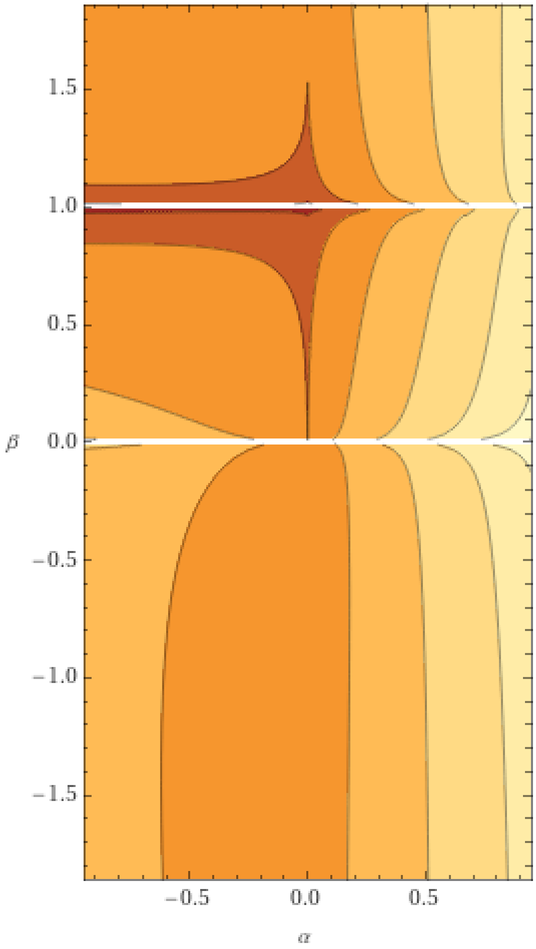


## Imaginary part

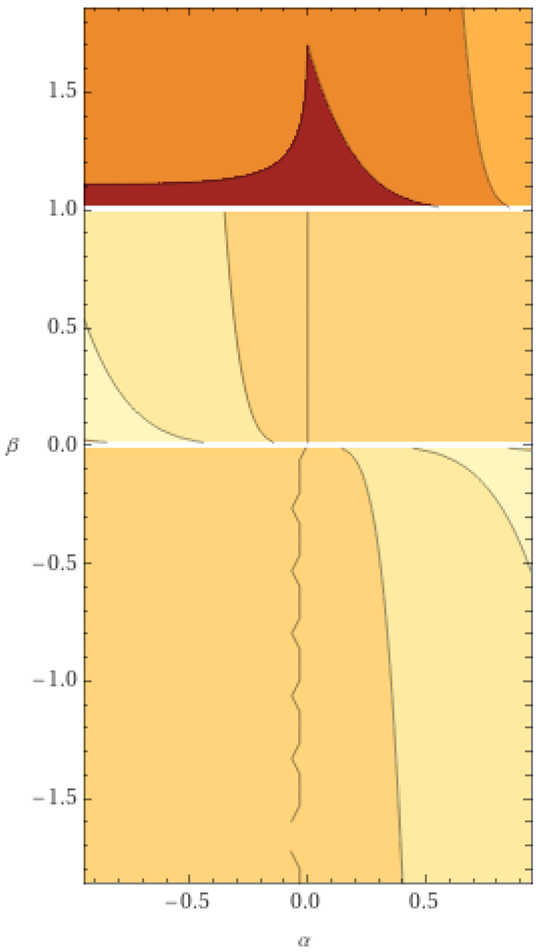


## Contour plots

### Real part



## Imaginary part



## Alternate form

$$\sqrt[8]{\frac{\alpha^5}{\beta}} - \sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}$$

## Alternate form assuming α and β are positive

$$\frac{\alpha^{5/8}}{\sqrt[8]{\beta}} - \frac{(1-\alpha)^{5/8}\exp\left(\frac{1}{4}\, i\, \pi \left\lfloor \frac{-5\arg(1-\alpha)+\arg(1-\beta)+\pi}{2\pi} \right\rfloor\right)}{\sqrt[8]{1-\beta}}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

## Series expansion at α=0

$$-\sqrt[8]{\frac{1}{1-\beta}} + \frac{\alpha^{5/8}\sqrt[8]{\frac{\alpha^5}{\beta}}}{\alpha^{5/8}} + \frac{5}{8}\alpha\sqrt[8]{\frac{1}{1-\beta}} +$$

$$\frac{15}{128}\alpha^2\sqrt[8]{\frac{1}{1-\beta}} + \frac{55\,\alpha^3\sqrt[8]{\frac{1}{1-\beta}}}{1024} + \frac{1045\,\alpha^4\sqrt[8]{\frac{1}{1-\beta}}}{32768} + O\left(\alpha^5\right)$$

(generalized Puiseux series)

## Series expansion at α=∞

$$\frac{\alpha^{5/8}\left(\sqrt[8]{\frac{\alpha^5}{\beta}} - \sqrt[8]{\frac{\alpha^5}{\beta-1}}\right)}{\alpha^{5/8}} + O\left(\sqrt[8]{\frac{1}{\alpha}}\right)$$

(Puiseux series)

## Derivative

$$\frac{\partial}{\partial\alpha}\left(\sqrt[8]{\frac{\alpha^5}{\beta}} - \sqrt[8]{\frac{(1-\alpha)^5}{1-\beta}}\right) = \frac{5}{8}\left(\frac{\sqrt[8]{\frac{\alpha^5}{\beta}}}{\alpha} + \frac{\sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}}{1-\alpha}\right)$$

## Indefinite integral

$$\int\left(\sqrt[8]{\frac{\alpha^5}{\beta}} - \sqrt[8]{\frac{(1-\alpha)^5}{1-\beta}}\right)d\alpha = \frac{8}{13}\left(\alpha\sqrt[8]{\frac{\alpha^5}{\beta}} - (\alpha-1)\sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}\right) + \text{constant}$$

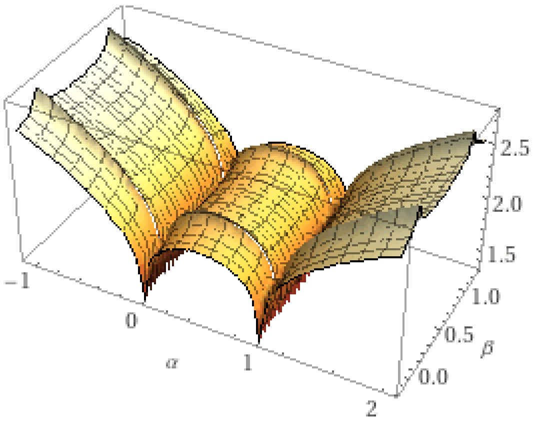
1+2^(1/3)*(((α^5(1-α)^5))/(β(1-β)))^(1/24)

## Input

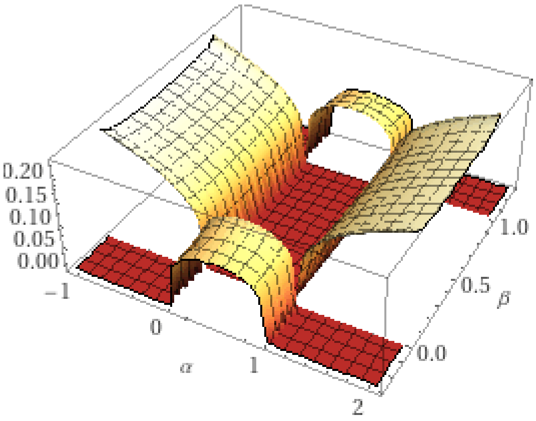
$$1 + \sqrt[3]{2}\,\sqrt[24]{\frac{\alpha^5\,(1-\alpha)^5}{\beta\,(1-\beta)}}$$

**3D plots** (figures that can be related to the D-branes/Instantons)

**Real part**

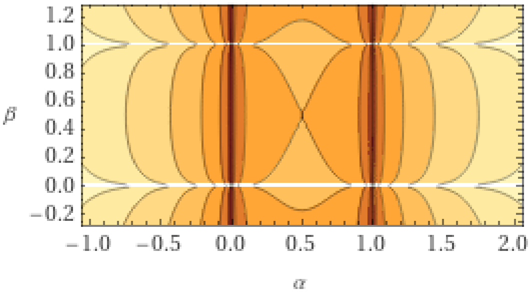


**Imaginary part**

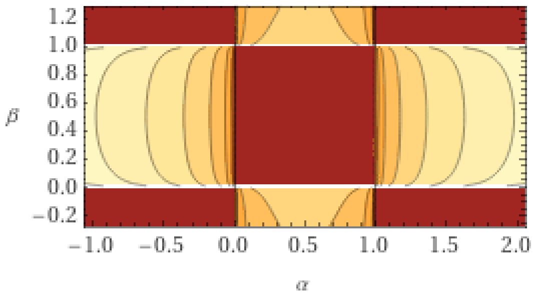


**Contour plots**

**Real part**



## Imaginary part



## Alternate form

$$\sqrt[3]{2}\,\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}+1$$

## Alternate forms assuming α and β are positive

$$\frac{\sqrt[24]{-(\beta-1)\,\beta}+\sqrt[3]{2}\,(-(\alpha-1)\,\alpha)^{5/24}\exp\left(\frac{1}{12}\,i\,\pi\left\lfloor\frac{-5\arg(1-\alpha)+\arg(1-\beta)+\pi}{2\,\pi}\right\rfloor\right)}{\sqrt[24]{-(\beta-1)\,\beta}}$$

$$1+\frac{\sqrt[3]{2}\,(1-\alpha)^{5/24}\,\alpha^{5/24}\exp\left(\frac{1}{12}\,i\,\pi\left\lfloor-\frac{5\arg(1-\alpha)}{2\,\pi}+\frac{\arg(1-\beta)}{2\,\pi}+\frac{1}{2}\right\rfloor\right)}{\sqrt[24]{1-\beta}\,\sqrt[24]{\beta}}$$

## Roots

(no roots exist)

## Series expansion at α=0

$$1+\frac{\sqrt[3]{2}\,\alpha^{5/24}\,\sqrt[24]{\frac{\alpha^5}{\beta-\beta^2}}}{\alpha^{5/24}}-\frac{5\,\alpha^{29/24}\,\sqrt[24]{\frac{\alpha^5}{\beta-\beta^2}}}{12\left(2^{2/3}\,\alpha^{5/24}\right)}-\frac{95\,\alpha^{53/24}\,\sqrt[24]{\frac{\alpha^5}{\beta-\beta^2}}}{576\left(2^{2/3}\,\alpha^{5/24}\right)}-\frac{4085\,\alpha^{77/24}\,\sqrt[24]{\frac{\alpha^5}{\beta-\beta^2}}}{41\,472\left(2^{2/3}\,\alpha^{5/24}\right)}-\frac{273\,695\,\alpha^{101/24}\,\sqrt[24]{\frac{\alpha^5}{\beta-\beta^2}}}{3\,981\,312\left(2^{2/3}\,\alpha^{5/24}\right)}+O\left(\alpha^5\right)$$

(generalized Puiseux series)

## Series expansion at α=∞

$$\frac{\sqrt[3]{2}\,\alpha^{5/12}\sqrt[24]{\frac{\alpha^{10}}{(\beta-1)\,\beta}}}{\alpha^{5/12}}+1+O\left(\sqrt[12]{\frac{1}{\alpha}}\right)$$

(Puiseux series)

## Derivative

$$\frac{\partial}{\partial\alpha}\left(1+\sqrt[3]{2}\,\sqrt[24]{\frac{\alpha^5\,(1-\alpha)^5}{\beta\,(1-\beta)}}\right)=\frac{5\,(2\,\alpha-1)\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}}{12\times 2^{2/3}\,(\alpha-1)\,\alpha}$$

## Indefinite integral

$$\int\left(1+\sqrt[3]{2}\,\sqrt[24]{\frac{\alpha^5\,(1-\alpha)^5}{\beta\,(1-\beta)}}\right)d\alpha=$$

$$\alpha+\frac{24\,\sqrt[3]{2}\,(\alpha-1)\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}\;{}_2F_1\left(-\frac{5}{24},\frac{29}{24};\frac{53}{24};1-\alpha\right)}{29\,\alpha^{5/24}}+\text{constant}$$

${}_2F_1(a,b;c;x)$ is the hypergeometric function

## Limit

$$\lim_{\beta\to\pm\infty}\left(1+\sqrt[3]{2}\,\sqrt[24]{\frac{(1-\alpha)^5\,\alpha^5}{(1-\beta)\,\beta}}\right)=1$$

For:

0.649335…= α

-573.259864339 = β

(0.649335^5/-573.259864339)^(1/8) – ((1-0.649335)^5 / (1+573.259864339))^(1/8)*(-5.2034 + 0.259455 i)

**Input interpretation**

$$\sqrt[8]{-\frac{0.649335^5}{573.259864339}} - \sqrt[8]{\frac{(1-0.649335)^5}{1+573.259864339}} \times (-5.2034 + 0.259455\, i)$$

$i$ is the imaginary unit

**Result**

$1.54055\ldots +$
$0.0711645\ldots\, i$

**Polar coordinates**

$r = 1.54219$ (radius), $\theta = 0.0461615$ (angle)

1.54219

1+2^(1/3)*(((0.649335^5(1-0.649335)^5)) / (-573.259864339(1+573.259864339)))^(1/24)

**Input interpretation**

$$1 + \sqrt[3]{2}\ \sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.259864339\,(1+573.259864339)}}$$

**Result**

$1.540548\ldots +$
$0.07116456\ldots\, i$

**Polar coordinates**

$r = 1.54219$ (radius), $\theta = 0.0461615$ (angle)

1.54219

From which:

1+1/((1+2^(1/3)*(((0.649335^5(1-0.649335)^5)) / (-573.259864339(1+573.259864339)))^(1/24)))

## Input interpretation

$$1+\frac{1}{1+\sqrt[3]{2}\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.259864339\,(1+573.259864339)}}}$$

## Result

$1.647737\ldots -$
$0.02992177\ldots\, i$

## Polar coordinates

$r = 1.64801$ (radius), $\theta = -0.0181573$ (angle)

$1.64801 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$ (trace of the instanton shape)

(((2Pi((1+2^(1/3)*(((0.649335^5(1-0.649335)^5)) / (-573.259864339(1+573.259864339)))^(1/24)))+(log(1/4 (-5 + e + e^2)))

where

$$\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right) \approx 0.2443839538$$

## Input interpretation

$$2\pi\left(1+\sqrt[3]{2}\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.259864339\,(1+573.259864339)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)$$

$\log(x)$ is the natural logarithm

## Result

$9.923936\ldots +$
$0.4471401\ldots\, i$

## Polar coordinates

$r = 9.934$ (radius), $\theta = 0.0450263$ (angle)

9.934 result very near to the entropy black hole ln(20619) = 9.933968….

## Polar forms

$$9.934\,(\cos(0.0450263) + i\sin(0.0450263))$$

---

$$9.934\,e^{0.0450263\,i}$$

## Alternative representations

$$2\pi\left(1+\sqrt[3]{2}\,\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.2598643390000\,(1+573.2598643390000)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)=$$
$$\log_e\left(\frac{1}{4}\left(-5+e+e^2\right)\right)+2\pi\left(1+\sqrt[3]{2}\,\sqrt[24]{-\frac{0.350665^5\times 0.649335^5}{329200.1319263077}}\right)$$

---

$$2\pi\left(1+\sqrt[3]{2}\,\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.2598643390000\,(1+573.2598643390000)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)=$$
$$\log(a)\log_a\left(\frac{1}{4}\left(-5+e+e^2\right)\right)+2\pi\left(1+\sqrt[3]{2}\,\sqrt[24]{-\frac{0.350665^5\times 0.649335^5}{329200.1319263077}}\right)$$

---

$$2\pi\left(1+\sqrt[3]{2}\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.2598643390000\,(1+573.2598643390000)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)=-\text{Li}_1\left(1-\frac{1}{4}\left(-5+e+e^2\right)\right)+2\pi\left(1+\sqrt[3]{2}\sqrt[24]{-\frac{0.350665^5\times 0.649335^5}{329200.1319263077}}\right)$$

## Series representations

$$2\pi\left(1+\sqrt[3]{2}\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.2598643390000\,(1+573.2598643390000)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)=(3.0811+0.142329\,i)\,\pi-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-9+e+e^2\right)^k}{k}$$

---

$$2\pi\left(1+\sqrt[3]{2}\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.2598643390000\,(1+573.2598643390000)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)=(3.0811+0.142329\,i)\,\pi+2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{1}{4}\left(-5+e+e^2-4\,x\right)\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-5+e+e^2-4\,x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

---

$$2\pi\left(1+\sqrt[3]{2}\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.2598643390000\,(1+573.2598643390000)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)=(3.0811+0.142329\,i)\,\pi+\left\lfloor\frac{\arg\left(\frac{1}{4}\left(-5+e+e^2-4\,z_0\right)\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{1}{4}\left(-5+e+e^2-4\,z_0\right)\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-5+e+e^2-4\,z_0\right)^k z_0^{-k}}{k}$$

## Integral representations

$$2\pi\left(1+\sqrt[3]{2}\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.2598643390000\,(1+573.2598643390000)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)=(3.0811+0.142329\,i)\,\pi+\int_1^{\frac{1}{4}\left(-5+e+e^2\right)}\frac{1}{t}\,dt$$

---

$$2\pi\left(1+\sqrt[3]{2}\sqrt[24]{-\frac{0.649335^5\,(1-0.649335)^5}{573.2598643390000\,(1+573.2598643390000)}}\right)+\log\left(\frac{1}{4}\left(-5+e+e^2\right)\right)=(3.0811+0.142329\,i)\,\pi+\frac{0.5}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^s\left(-9+e+e^2\right)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

From:

$r = 9.934$ (radius), $\theta = 0.0450263$ (angle)

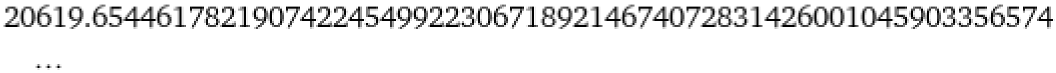
we obtain also:

exp(9.934)

**Input**

exp(9.934)

**Result**



20619.65446178219074224549922306718921467407283142600104590335657 4

…

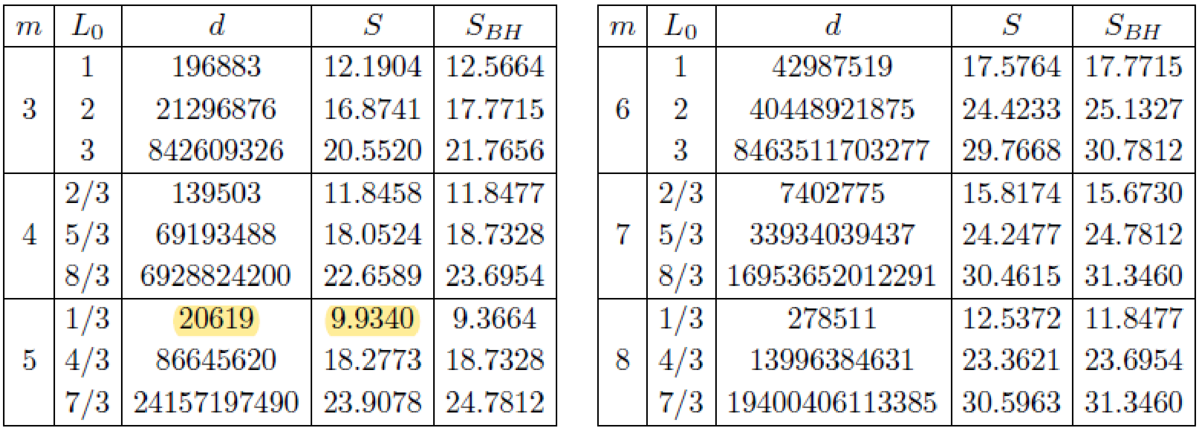
**20619**.6544617….

From:

**Three-dimensional AdS gravity and extremal CFTs at c = 8m**

*Spyros D. Avramis, Alex Kehagiasb and Constantina Mattheopoulou*

Received: September 7, 2007 -Accepted: October 28, 2007 - Published: November 9, 2007



| $m$ | $L_0$ | $d$ | $S$ | $S_{BH}$ |
|---|---|---|---|---|
| 3 | 1 | 196883 | 12.1904 | 12.5664 |
|  | 2 | 21296876 | 16.8741 | 17.7715 |
|  | 3 | 842609326 | 20.5520 | 21.7656 |
| 4 | 2/3 | 139503 | 11.8458 | 11.8477 |
|  | 5/3 | 69193488 | 18.0524 | 18.7328 |
|  | 8/3 | 6928824200 | 22.6589 | 23.6954 |
| 5 | 1/3 | 20619 | 9.9340 | 9.3664 |
|  | 4/3 | 86645620 | 18.2773 | 18.7328 |
|  | 7/3 | 24157197490 | 23.9078 | 24.7812 |

| $m$ | $L_0$ | $d$ | $S$ | $S_{BH}$ |
|---|---|---|---|---|
| 6 | 1 | 42987519 | 17.5764 | 17.7715 |
|  | 2 | 40448921875 | 24.4233 | 25.1327 |
|  | 3 | 8463511703277 | 29.7668 | 30.7812 |
| 7 | 2/3 | 7402775 | 15.8174 | 15.6730 |
|  | 5/3 | 33934039437 | 24.2477 | 24.7812 |
|  | 8/3 | 16953652012291 | 30.4615 | 31.3460 |
| 8 | 1/3 | 278511 | 12.5372 | 11.8477 |
|  | 4/3 | 13996384631 | 23.3621 | 23.6954 |
|  | 7/3 | 19400406113385 | 30.5963 | 31.3460 |

**Table 1:** Degeneracies, microscopic entropies and semiclassical entropies for the first few values of $m$ and $L_0$.

Now, we consider the two alternate forms of the two previous equations:

$$\sqrt[8]{\frac{\alpha^5}{\beta}} - \sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}$$

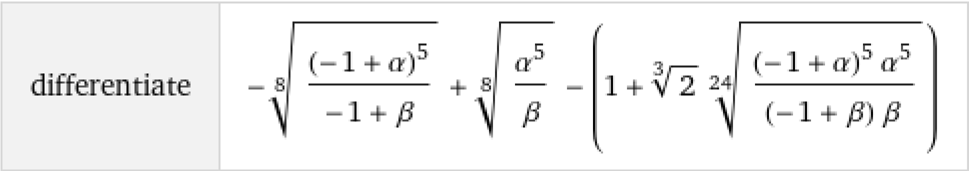
and

$$\sqrt[3]{2}\,\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}+1$$

Subtracting the first to the second and perform the derivative, we obtain:

derivative[-((-1 + α)^5/(-1 + β))^(1/8) + (α^5/β)^(1/8) - ((1 + 2^(1/3) (((-1 + α)^5 α^5)/((-1 + β) β))^(1/24)))]

**Input interpretation**



| differentiate | $-\sqrt[8]{\frac{(-1+\alpha)^5}{-1+\beta}}+\sqrt[8]{\frac{\alpha^5}{\beta}}-\left(1+\sqrt[3]{2}\,\sqrt[24]{\frac{(-1+\alpha)^5\,\alpha^5}{(-1+\beta)\,\beta}}\right)$ |
|---|---|

**Partial derivatives**

$$\frac{\partial}{\partial\alpha}\left(\sqrt[8]{\frac{\alpha^5}{\beta}}-\sqrt[3]{2}\,\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}-\sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}-1\right)=\\ \frac{1}{24}\left(-\frac{5\sqrt[3]{2}\,(2\,\alpha-1)\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}}{(\alpha-1)\,\alpha}+\frac{15\sqrt[8]{\frac{\alpha^5}{\beta}}}{\alpha}-\frac{15\sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}}{\alpha-1}\right)$$

---

$$\frac{\partial}{\partial\beta}\left(\sqrt[8]{\frac{\alpha^5}{\beta}}-\sqrt[3]{2}\,\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}-\sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}-1\right)=\\ \frac{1}{24}\left(\frac{\sqrt[3]{2}\,(2\,\beta-1)\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}}{(\beta-1)\,\beta}-\frac{3\sqrt[8]{\frac{\alpha^5}{\beta}}}{\beta}+\frac{3\sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}}{\beta-1}\right)$$

**Differential**

$$d\left(-1-\sqrt[8]{\frac{(-1+\alpha)^5}{-1+\beta}}+\sqrt[8]{\frac{\alpha^5}{\beta}}-\sqrt[3]{2}\sqrt[24]{\frac{(-1+\alpha)^5\,\alpha^5}{(-1+\beta)\,\beta}}\right)=$$

$$\frac{1}{24}\left(-\frac{15\sqrt[8]{\frac{(-1+\alpha)^5}{-1+\beta}}}{-1+\alpha}+\frac{15\sqrt[8]{\frac{\alpha^5}{\beta}}}{\alpha}-\frac{5\sqrt[3]{2}\,(-1+2\,\alpha)\sqrt[24]{\frac{(-1+\alpha)^5\,\alpha^5}{(-1+\beta)\,\beta}}}{(-1+\alpha)\,\alpha}\right)d\alpha+$$

$$\frac{1}{24}\left(\frac{3\sqrt[8]{\frac{(-1+\alpha)^5}{-1+\beta}}}{-1+\beta}-\frac{3\sqrt[8]{\frac{\alpha^5}{\beta}}}{\beta}+\frac{\sqrt[3]{2}\sqrt[24]{\frac{(-1+\alpha)^5\,\alpha^5}{(-1+\beta)\,\beta}}\,(-1+2\,\beta)}{(-1+\beta)\,\beta}\right)d\beta$$

From

$$\frac{\partial}{\partial\alpha}\left(\sqrt[8]{\frac{\alpha^5}{\beta}}-\sqrt[3]{2}\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}-\sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}-1\right)=$$

$$\frac{1}{24}\left(-\frac{5\sqrt[3]{2}\,(2\,\alpha-1)\sqrt[24]{\frac{(\alpha-1)^5\,\alpha^5}{(\beta-1)\,\beta}}}{(\alpha-1)\,\alpha}+\frac{15\sqrt[8]{\frac{\alpha^5}{\beta}}}{\alpha}-\frac{15\sqrt[8]{\frac{(\alpha-1)^5}{\beta-1}}}{\alpha-1}\right)$$

for α = β = 1/2 , and multiplying by 4, we obtain:

4*1/24 ((2^(1/3) (2*0.5 - 1) (((0.5 - 1)^5 0.5^5)/((0.5 - 1) 0.5))^(1/24))/((0.5 - 1) 0.5) - (3 (0.5^5/0.5)^(1/8))/0.5 + (3 ((0.5- 1)^5/(0.5 - 1))^(1/8))/(0.5 - 1))

### Input

$$4\times\frac{1}{24}\left(\frac{\sqrt[3]{2}\,(2\times0.5-1)\sqrt[24]{\frac{(0.5-1)^5\times0.5^5}{(0.5-1)\times0.5}}}{(0.5-1)\times0.5}-\frac{3\sqrt[8]{\frac{0.5^5}{0.5}}}{0.5}+\frac{3\sqrt[8]{\frac{(0.5-1)^5}{0.5-1}}}{0.5-1}\right)$$

### Result

$-1.4142135623730950488016887242096980785696718753769480731766797 37\ldots$

-1.414213562373….

### Possible closed forms

$-\sqrt{2}\approx$
$-1.414213562373095048801688724209698078569671875376948073176679737$
99073

---

$-e^{-1/8\,b_4(2)}\approx$
$-1.414213562373095048801688724209698078569671875376948073176679737$
99073

---

$-\tan\left(\frac{\theta_t}{2}\right)\approx$
$-1.414213562373095048801688724209698078569671875376948073176679737$
99073

Or also:

sqrt((-0.9568666373/(((1/24 ((2^(1/3) (2*0.5 - 1) (((0.5 - 1)^5 0.5^5)/((0.5 - 1) 0.5))^(1/24))/((0.5 - 1) 0.5) - (3 (0.5^5/0.5)^(1/8))/0.5 + (3 ((0.5- 1)^5/(0.5 - 1))^(1/8))/(0.5 - 1)))))))

**Input interpretation**

$$\sqrt{-\frac{0.9568666373}{\frac{1}{24}\left(\frac{\sqrt[3]{2}\,(2\times 0.5-1)\sqrt[24]{\frac{(0.5-1)^5\times 0.5^5}{(0.5-1)\times 0.5}}}{(0.5-1)\times 0.5}-\frac{3\sqrt[8]{\frac{0.5^5}{0.5}}}{0.5}+\frac{3\sqrt[8]{\frac{(0.5-1)^5}{0.5-1}}}{0.5-1}\right)}}$$

**Result**

1.6451223515909066070828466757295792329364604330541111621354108231

…

1.64512235159….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

And again:

((((2^(1/3) (2*0.5 - 1) (((0.5 - 1)^5 0.5^5)/((0.5 - 1) 0.5))^(1/24))/((0.5 - 1) 0.5) - (3 (0.5^5/0.5)^(1/8))/0.5 + (3 ((0.5- 1)^5/(0.5 - 1))^(1/8))/(0.5 - 1)))) 1/(-0.3535533905932)

**Input interpretation**

$$\left(\frac{\sqrt[3]{2}\,(2\times 0.5-1)\sqrt[24]{\frac{(0.5-1)^5\times 0.5^5}{(0.5-1)\times 0.5}}}{(0.5-1)\times 0.5}-\frac{3\sqrt[8]{\frac{0.5^5}{0.5}}}{0.5}+\frac{3\sqrt[8]{\frac{(0.5-1)^5}{0.5-1}}}{0.5-1}\right)\left(-\frac{1}{0.3535533905932}\right)$$

**Result**

24.0000…

24

From which:

72*((((2^(1/3) (2*0.5 - 1) (((0.5 - 1)^5 0.5^5)/((0.5 - 1) 0.5))^(1/24))/((0.5 - 1) 0.5) - (3 (0.5^5/0.5)^(1/8))/0.5 + (3 ((0.5- 1)^5/(0.5 - 1))^(1/8))/(0.5 - 1)))) 1/(-0.3535533905932)+1

**Input interpretation**

$$72\left(\frac{\sqrt[3]{2}\,(2\times 0.5-1)\sqrt[24]{\frac{(0.5-1)^5\times 0.5^5}{(0.5-1)\times 0.5}}}{(0.5-1)\times 0.5}-\frac{3\sqrt[8]{\frac{0.5^5}{0.5}}}{0.5}+\frac{3\sqrt[8]{\frac{(0.5-1)^5}{0.5-1}}}{0.5-1}\right)\left(-\frac{1}{0.3535533905932}\right)+1$$

**Result**

1729.00…

1729

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(1/27((72*((((2^(1/3) (2*0.5 - 1) (((0.5 - 1)^5 0.5^5)/((0.5 - 1) 0.5))^(1/24))/((0.5 - 1) 0.5) - (3 (0.5^5/0.5)^(1/8))/0.5 + (3 ((0.5- 1)^5/(0.5 - 1))^(1/8))/(0.5 - 1)))) 1/(-0.3535533905932))))^2

**Input interpretation**

$$\left(\frac{1}{27}\left(72\left(\frac{\sqrt[3]{2}\,(2\times 0.5-1)\sqrt[24]{\frac{(0.5-1)^5\times 0.5^5}{(0.5-1)\times 0.5}}}{(0.5-1)\times 0.5}-\frac{3\sqrt[8]{\frac{0.5^5}{0.5}}}{0.5}+\frac{3\sqrt[8]{\frac{(0.5-1)^5}{0.5-1}}}{0.5-1}\right)\left(-\frac{1}{0.3535533905932}\right)\right)\right)^2$$

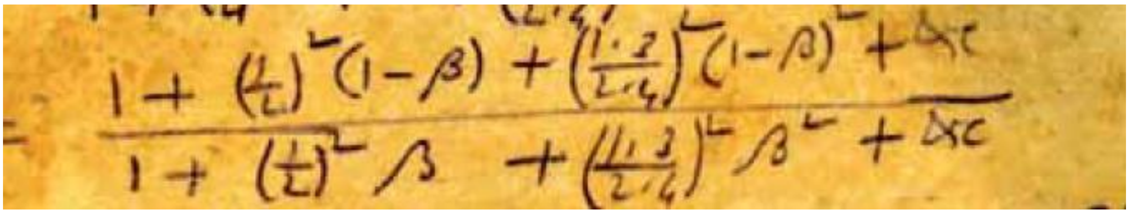
## Result

4096.000000001709105221264400824691736332436911242383148527179234 0

…

$\approx 4096 = 64^2$

From:



$$= \frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{1\cdot 3}{2\cdot 4}\right)^2(1-\beta)^2+\&c}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\beta^2+\&c}$$

(((1+(1/2)^2 (1-β)+ ((1*3)/(2*4))^2 (1-β)^2))) / (((1+(1/2)^2 * β + ((1*3)/(2*4))^2 β^2)))

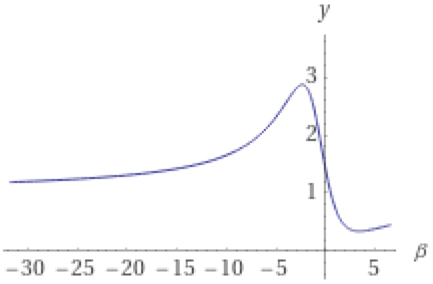
## Input

$$\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{1\times 3}{2\times 4}\right)^2(1-\beta)^2}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{1\times 3}{2\times 4}\right)^2\beta^2}$$

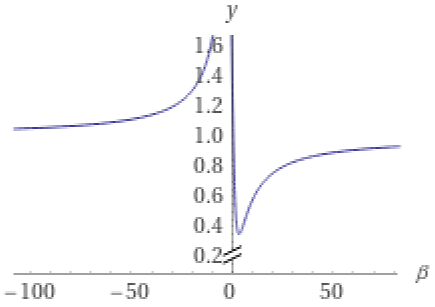
## Result

$$\frac{\frac{9}{64}(1-\beta)^2+\frac{1-\beta}{4}+1}{\frac{9\beta^2}{64}+\frac{\beta}{4}+1}$$

## Plots



(β from −31.7 to 6.5)



(β from −108 to 82.8)

## Alternate forms

$$\frac{9\beta^2 - 34\beta + 89}{9\beta^2 + 16\beta + 64}$$

$$\frac{25 - 50\beta}{\beta(9\beta + 16) + 64} + 1$$

$$1 - \frac{25(2\beta - 1)}{9\beta^2 + 16\beta + 64}$$

## Expanded forms

$$\frac{9\beta^2}{64\left(\frac{9\beta^2}{64} + \frac{\beta}{4} + 1\right)} - \frac{17\beta}{32\left(\frac{9\beta^2}{64} + \frac{\beta}{4} + 1\right)} + \frac{89}{64\left(\frac{9\beta^2}{64} + \frac{\beta}{4} + 1\right)}$$

$$\frac{9\beta^2}{64\left(\frac{9\beta^2}{64} + \frac{\beta}{4} + 1\right)} - \frac{17\beta}{32\left(\frac{9\beta^2}{64} + \frac{\beta}{4} + 1\right)} + \frac{89}{9\beta^2 + 16\beta + 64}$$

## Complex roots

$$\beta = \frac{17}{9} - \frac{16i\sqrt{2}}{9}$$

$$\beta = \frac{17}{9} + \frac{16\, i \sqrt{2}}{9}$$

## Series expansion at β=0

$$\frac{89}{64} - \frac{225\,\beta}{256} + \frac{675\,\beta^2}{4096} + \frac{675\,\beta^3}{8192} - \frac{11\,475\,\beta^4}{262\,144} + O\left(\beta^5\right)$$

(Taylor series)

## Series expansion at β=∞

$$1 - \frac{50}{9\,\beta} + \frac{1025}{81\,\beta^2} + \frac{12\,400}{729\,\beta^3} + O\left(\left(\frac{1}{\beta}\right)^4\right)$$

(Laurent series)

## Derivative

$$\frac{d}{d\beta}\left(\frac{1 + \left(\frac{1}{2}\right)^2 (1-\beta) + \left(\frac{3}{2\times 4}\right)^2 (1-\beta)^2}{1 + \left(\frac{1}{2}\right)^2 \beta + \left(\frac{3}{2\times 4}\right)^2 \beta^2}\right) = \frac{450\left(\beta^2 - \beta - 8\right)}{\left(9\,\beta^2 + 16\,\beta + 64\right)^2}$$

## Indefinite integral

$$\int \frac{1 + \frac{1-\beta}{4} + \frac{9}{64}\,(1-\beta)^2}{1 + \frac{\beta}{4} + \frac{9\,\beta^2}{64}}\, d\beta =$$

$$-\frac{25}{9}\log\left(9\,\beta^2 + 16\,\beta + 64\right) + \beta + \frac{625\tan^{-1}\left(\frac{9\,\beta+8}{16\sqrt{2}}\right)}{144\sqrt{2}} + \text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

## Global maximum

$$\max\left\{\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{3}{2\times 4}\right)^2(1-\beta)^2}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{3}{2\times 4}\right)^2\beta^2}\right\}=\frac{1649+225\sqrt{33}}{1024}\quad\text{at}\quad\beta=\frac{1}{2}-\frac{\sqrt{33}}{2}$$

## Global minimum

$$\min\left\{\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{3}{2\times 4}\right)^2(1-\beta)^2}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{3}{2\times 4}\right)^2\beta^2}\right\}=\frac{1649-225\sqrt{33}}{1024}\quad\text{at}\quad\beta=\frac{1}{2}+\frac{\sqrt{33}}{2}$$

## Limit

$$\lim_{\beta\to\pm\infty}\frac{1+\frac{1-\beta}{4}+\frac{9}{64}(1-\beta)^2}{1+\frac{\beta}{4}+\frac{9\beta^2}{64}}=1$$

## Series representations

$$\frac{1+\frac{1-\beta}{4}+\frac{9}{64}(1-\beta)^2}{1+\frac{\beta}{4}+\frac{9\beta^2}{64}}=\frac{89}{64}+\sum_{n=1}^{\infty}\beta^n\,25\left(2^{-17/2-3n}\left(\left(-1-2i\sqrt{2}\right)^n\left(-17i+2\sqrt{2}\right)+\left(-1+2i\sqrt{2}\right)^n\left(17i+2\sqrt{2}\right)\right)\right)$$

---

$$\frac{1+\frac{1-\beta}{4}+\frac{9}{64}(1-\beta)^2}{1+\frac{\beta}{4}+\frac{9\beta^2}{64}}=\frac{64}{89}+\sum_{n=1}^{\infty}\frac{1}{32\sqrt{2}}(-1+\beta)^n(-25)\left(89^{-1-n}\left(\left(-17+16i\sqrt{2}\right)^n\left(-161i+16\sqrt{2}\right)+\left(-17-16i\sqrt{2}\right)^n\left(161i+16\sqrt{2}\right)\right)\right)$$

From:

$$\frac{\frac{9}{64}(1-\beta)^2+\frac{1-\beta}{4}+1}{\frac{9\beta^2}{64}+\frac{\beta}{4}+1}$$

for β = 6.5 :

(1 + (1 - 6.5)/4 + 9/64 (1 - 6.5)^2)/(1 + 6.5/4 + (9 6.5^2)/64)

**Input**

$$\frac{1+\frac{1-6.5}{4}+\frac{9}{64}(1-6.5)^2}{1+\frac{6.5}{4}+\frac{1}{64}\left(9\times 6.5^2\right)}$$

**Result**

0.4528043775649794801641586867305061559507523939808481532147742818

…

0.45280437756497…..

for β = 82.8 :

(1 + (1 - 82.8)/4 + 9/64 (1 - 82.8)^2)/(1 + 82.8/4 + (9 82.8^2)/64)

**Input**

$$\frac{1+\frac{1-82.8}{4}+\frac{9}{64}(1-82.8)^2}{1+\frac{82.8}{4}+\frac{1}{64}\left(9\times 82.8^2\right)}$$

**Result**

0.9347771232067275138782869793898879339421435835271263767336763702

…

0.9347771232067275……

From the sum of the two expressions, after some calculations, we obtain:

sqrt[2*((0.9568666373+0.9991104684)/2)(((1 + (1 - 6.5)/4 + 9/64 (1 - 6.5)^2)/(1 + 6.5/4 + (9 6.5^2)/64) + (1 + (1 - 82.8)/4 + 9/64 (1 - 82.8)^2)/(1 + 82.8/4 + (9 82.8^2)/64)))]

where 0.9568666373 and 0.9991104684 are two values of the following Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\dots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

**Input interpretation**

$$\sqrt{\left(2\times\frac{0.9568666373+0.9991104684}{2}\left(\frac{1+\frac{1-6.5}{4}+\frac{9}{64}(1-6.5)^2}{1+\frac{6.5}{4}+\frac{1}{64}\left(9\times 6.5^2\right)}+\frac{1+\frac{1-82.8}{4}+\frac{9}{64}(1-82.8)^2}{1+\frac{82.8}{4}+\frac{1}{64}\left(9\times 82.8^2\right)}\right)\right)}$$

**Result**

1.6474457951029240957955149739759275771948558792857440354507635127…

1.6474457951…. ≈ $\zeta(2)=\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

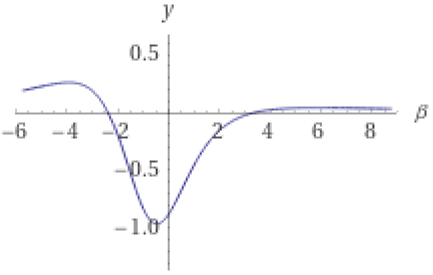
We perform the following calculations:

derivative(((1+(1/2)^2 (1-β)+ ((1*3)/(2*4))^2 (1-β)^2))) / (((1+(1/2)^2 * β + ((1*3)/(2*4))^2 β^2)))

**Derivative**

$$\frac{d}{d\beta}\left(\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{3}{2\times 4}\right)^2(1-\beta)^2}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{3}{2\times 4}\right)^2\beta^2}\right)=\frac{450\left(\beta^2-\beta-8\right)}{\left(9\beta^2+16\beta+64\right)^2}$$

**Plots**



(β from −5.7 to 8.7)

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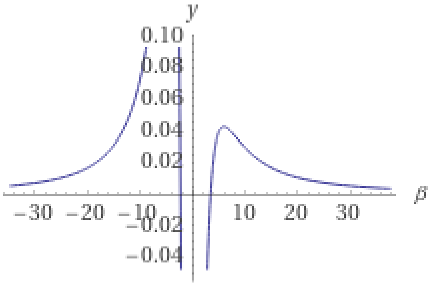


(β from −34.6 to 37.6)

**Alternate forms**

$$\frac{450((\beta-1)\beta-8)}{(\beta(9\beta+16)+64)^2}$$

---

$$\frac{\beta(450\beta-450)-3600}{\beta(\beta(\beta(81\beta+288)+1408)+2048)+4096}$$

**Partial fraction expansion**

$$\frac{50}{9\beta^2+16\beta+64}-\frac{50(25\beta+136)}{\left(9\beta^2+16\beta+64\right)^2}$$

## Expanded forms

$$\frac{450\beta^2}{81\beta^4+288\beta^3+1408\beta^2+2048\beta+4096}-\frac{450\beta}{81\beta^4+288\beta^3+1408\beta^2+2048\beta+4096}-\frac{3600}{81\beta^4+288\beta^3+1408\beta^2+2048\beta+4096}$$

$$\frac{450\beta^2-450\beta-3600}{81\beta^4+288\beta^3+1408\beta^2+2048\beta+4096}$$

## Roots

$$\beta=\frac{1}{2}-\frac{\sqrt{33}}{2}$$

$$\beta=\frac{1}{2}+\frac{\sqrt{33}}{2}$$

## Series expansion at β=0

$$-\frac{225}{256}+\frac{675\beta}{2048}+\frac{2025\beta^2}{8192}-\frac{11475\beta^3}{65536}-\frac{3375\beta^4}{1048576}+O\left(\beta^5\right)$$

(Taylor series)

## Series expansion at β=∞

$$\frac{50}{9\,\beta^2} - \frac{2050}{81\,\beta^3} - \frac{12\,400}{243\,\beta^4} + \frac{3\,155\,200}{6561\,\beta^5} + O\left(\left(\frac{1}{\beta}\right)^6\right)$$

(Laurent series)

## Indefinite integral

$$\int \frac{450\left(-8-\beta+\beta^2\right)}{\left(64+16\,\beta+9\,\beta^2\right)^2}\,d\beta = \frac{25-50\,\beta}{9\,\beta^2+16\,\beta+64} + \text{constant}$$

## Local maxima

$$\max\left\{\frac{450\left(\beta^2-\beta-8\right)}{\left(9\,\beta^2+16\,\beta+64\right)^2}\right\} \approx 0.26070 \text{ at } \beta \approx -3.9238$$

---

$$\max\left\{\frac{450\left(\beta^2-\beta-8\right)}{\left(9\,\beta^2+16\,\beta+64\right)^2}\right\} \approx 0.042292 \text{ at } \beta \approx 5.8856$$

## Local minimum

$$\min\left\{\frac{450\left(\beta^2-\beta-8\right)}{\left(9\,\beta^2+16\,\beta+64\right)^2}\right\} \approx -0.96217 \text{ at } \beta \approx -0.46188$$

## Limit

$$\lim_{\beta\to\pm\infty} \frac{450\left(-8-\beta+\beta^2\right)}{\left(64+16\,\beta+9\,\beta^2\right)^2} = 0$$

## Definite integral

$$\int_0^\infty \frac{450\left(-8-\beta+\beta^2\right)}{\left(64+16\,\beta+9\,\beta^2\right)^2}\,d\beta = -\frac{25}{64} = -0.390625$$

From:

$$\frac{d}{d\beta}\left(\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{3}{2\times 4}\right)^2(1-\beta)^2}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{3}{2\times 4}\right)^2\beta^2}\right) = \frac{450\left(\beta^2-\beta-8\right)}{\left(9\,\beta^2+16\,\beta+64\right)^2}$$

we obtain, for β = 8.7 and 37.6 :

(450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2

**Input**

$$\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times 8.7+9\times 8.7^2\right)^2}$$

**Result**

0.0339377949190686866502717045312159360154663404711556830674890395

…

0.033937794919….

(450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2

**Input**

$$\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times 37.6+9\times 37.6^2\right)^2}$$

**Result**

0.0034341920297328709820193684390072521962041927464073680700048781
...

0.00343419202….

From which:

((((450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2 + (450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2)))^48

**Input**

$$\left(\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times 8.7+9\times 8.7^2\right)^2}+\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times 37.6+9\times 37.6^2\right)^2}\right)^{48}$$

**Result**

$3.0354078186632568216366107605966769350712011813380301580071\ldots \times 10^{-69}$

$3.035407818\ldots * 10^{-69}$

Now, we have:

integrate(((1+(1/2)^2 (1-β)+ ((1*3)/(2*4))^2 (1-β)^2))) / (((1+(1/2)^2 * β + ((1*3)/(2*4))^2 β^2)))

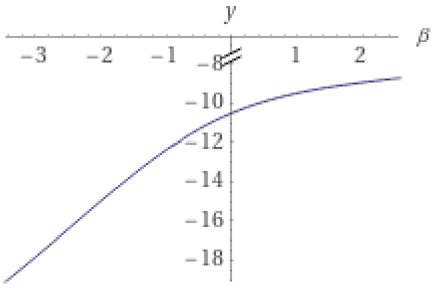
**Indefinite integral**

$$\int \frac{1+\left(\frac{1}{2}\right)^2 (1-\beta)+\left(\frac{3}{2\times 4}\right)^2 (1-\beta)^2}{1+\left(\frac{1}{2}\right)^2 \beta+\left(\frac{3}{2\times 4}\right)^2 \beta^2}\, d\beta = -\frac{25}{9}\log\left(9\beta^2+16\beta+64\right)+\beta+\frac{625\tan^{-1}\left(\frac{9\beta+8}{16\sqrt{2}}\right)}{144\sqrt{2}} + \text{constant}$$

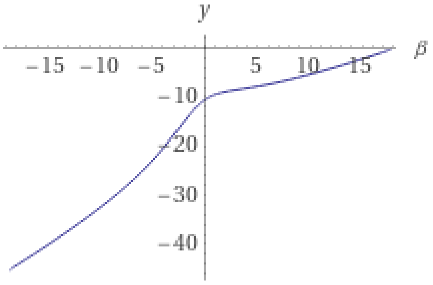
$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

## Plots of the integral



(β from −3.5 to 2.6)



(β from −18.5 to 17.6)

## Alternate forms of the integral

$$\beta - \frac{25}{9}\log(\beta\,(9\,\beta + 16) + 64) + \frac{625\tan^{-1}\left(\frac{9\,\beta+8}{16\sqrt{2}}\right)}{144\sqrt{2}} + \text{constant}$$

$$\frac{25}{288}\left(25\sqrt{2}\,\tan^{-1}\left(\frac{9\,\beta + 8}{16\sqrt{2}}\right) - 32\log\left(9\,\beta^2 + 16\,\beta + 64\right)\right) + \beta + \text{constant}$$

$$\frac{1}{288}\left(-800\log\left(9\,\beta^2 + 16\,\beta + 64\right) + 288\,\beta + 625\sqrt{2}\,\tan^{-1}\left(\frac{9\,\beta + 8}{16\sqrt{2}}\right)\right) + \text{constant}$$

## Series expansion of the integral at β=0

$$\frac{25}{288}\left(25\sqrt{2}\,\tan^{-1}\left(\frac{1}{2\sqrt{2}}\right) - 192\log(2)\right) + \frac{89\,\beta}{64} - \frac{225\,\beta^2}{512} + \frac{225\,\beta^3}{4096} + O\left(\beta^4\right)$$

(Taylor series)

**Series expansion of the integral at β=1/9 (-8-16 i sqrt(2))**

$$\left(\frac{1}{1152}\left(1250\,i\sqrt{2}\,\log\left(\beta+\frac{1}{9}\left(8+16\,i\sqrt{2}\right)\right)-\right.\right.$$
$$3200\log\left(-32\,i\sqrt{2}\left(\beta+\frac{1}{9}\left(8+16\,i\sqrt{2}\right)\right)\right)+2500\,i\sqrt{2}\,\log(3)-$$
$$\left.6875\,i\sqrt{2}\,\log(2)+625\sqrt{2}\,\pi-2048\,i\sqrt{2}-1024\right)+$$
$$\frac{\left(2673-800\,i\sqrt{2}\right)\left(\beta-\frac{1}{9}\left(-8-16\,i\sqrt{2}\right)\right)}{2048}-$$
$$\frac{225\,i\left(-64\,i+25\sqrt{2}\right)\left(\beta-\frac{1}{9}\left(-8-16\,i\sqrt{2}\right)\right)^2}{262144}+$$
$$\frac{675\,i\left(25\,i+32\sqrt{2}\right)\left(\beta-\frac{1}{9}\left(-8-16\,i\sqrt{2}\right)\right)^3}{4194304}+$$
$$\left.O\left(\left(\beta-\frac{1}{9}\left(-8-16\,i\sqrt{2}\right)\right)^4\right)\right)-$$
$$\frac{625\,\pi\left\lfloor\frac{-\arg\left(9\,\beta+16\,i\sqrt{2}+8\right)-\arg\left(-\frac{9\sqrt{2}\,\beta+8\sqrt{2}+32\,i}{-9\,i\,\beta+16\sqrt{2}-8\,i}\right)+\pi}{2\pi}\right\rfloor}{144\sqrt{2}}$$

**Series expansion of the integral at β=8/9 (-1+2 i sqrt(2))**

$$\left(\frac{1}{1152}\left(-1250\,i\sqrt{2}\,\log\left(\beta-\frac{8}{9}\left(-1+2\,i\sqrt{2}\right)\right)-\right.\right.$$
$$3200\log\left(32\,i\sqrt{2}\left(\beta-\frac{8}{9}\left(-1+2\,i\sqrt{2}\right)\right)\right)-2500\,i\sqrt{2}\,\log(3)+$$
$$\left.6875\,i\sqrt{2}\,\log(2)+625\sqrt{2}\,\pi+2048\,i\sqrt{2}-1024\right)+$$
$$\frac{\left(2673+800\,i\sqrt{2}\right)\left(\beta-\frac{8}{9}\left(-1+2\,i\sqrt{2}\right)\right)}{2048}+$$
$$\frac{225\,i\left(64\,i+25\sqrt{2}\right)\left(\beta-\frac{8}{9}\left(-1+2\,i\sqrt{2}\right)\right)^2}{262144}-$$
$$\left.\frac{675\,i\left(-25\,i+32\sqrt{2}\right)\left(\beta-\frac{8}{9}\left(-1+2\,i\sqrt{2}\right)\right)^3}{4194304}+O\left(\left(\beta-\frac{8}{9}\left(-1+2\,i\sqrt{2}\right)\right)^4\right)\right)+$$
$$\frac{625\,\pi\left\lfloor\frac{-\arg\left(9\,\beta-16\,i\sqrt{2}+8\right)-\arg\left(-\frac{9\sqrt{2}\,\beta+8\sqrt{2}-32\,i}{9\,i\,\beta+16\sqrt{2}+8\,i}\right)+\pi}{2\pi}\right\rfloor}{144\sqrt{2}}$$

From:

$$\beta - \frac{25}{9} \log(\beta(9\beta + 16) + 64) + \frac{625 \tan^{-1}\left(\frac{9\beta+8}{16\sqrt{2}}\right)}{144\sqrt{2}} + \text{constant}$$

For β = 2.6 and 17.6 :

2.6 -25/9 ln(2.6(9*2.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*2.6+8)/(16sqrt2)))))

## Input

$$2.6 - \frac{25}{9} \log(2.6(9\times 2.6 + 16) + 64) + \frac{1}{144\sqrt{2}}\left(625 \tan^{-1}\left(\frac{9\times 2.6 + 8}{16\sqrt{2}}\right)\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

## Result

−8.7028837321606294744104016914731793465464375968243312965917745 28…

(result in radians)

-8.70288373216….

## Alternative representations

$$2.6 - \frac{1}{9} \log(2.6(9\times 2.6 + 16) + 64)\, 25 + \frac{625 \tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}} =$$

$$2.6 - \frac{25 \log(166.44)}{9} + \frac{625 \tan^{-1}\left(1, \frac{31.4}{16\sqrt{2}}\right)}{144\sqrt{2}}$$

---

$$2.6 - \frac{1}{9} \log(2.6\,(9 \times 2.6 + 16) + 64)\,25 + \frac{625 \tan^{-1}\left(\frac{9 \times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}} =$$

$$2.6 - \frac{25 \log_e(166.44)}{9} + \frac{625 \tan^{-1}\left(\frac{31.4}{16\sqrt{2}}\right)}{144\sqrt{2}}$$

---

$$2.6 - \frac{1}{9} \log(2.6\,(9 \times 2.6 + 16) + 64)\,25 + \frac{625 \tan^{-1}\left(\frac{9 \times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}} =$$

$$2.6 - \frac{25}{9} \log(a) \log_a(166.44) + \frac{625 \tan^{-1}\left(\frac{31.4}{16\sqrt{2}}\right)}{144\sqrt{2}}$$

## Series representations

$$2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$-\Bigg(\Bigg(2.77778\Bigg(-0.936\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+$$

$$\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\log(165.44)\,\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-$$

$$1.5625\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k 3.925^{1+2k}\,F_{1+2k}\left(\frac{1}{\sqrt{2}+\sqrt{2}\,\sqrt{1+\frac{3.08113}{\sqrt{2}^{\,2}}}}\right)^{1+2k}}{1+2k}-$$

$$\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\Bigg(\sqrt{x}\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,(-0.00604449)^{k_1}\,(2-x)^{k_2}\,x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}}{k_2!\,k_1}\Bigg)\Bigg)\Bigg)$$

$$\Bigg/\Bigg(\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\Bigg)\quad\text{for }(x\in\mathbb{R}\text{ and }x<0)$$

$$
\begin{aligned}
&2.6 - \frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25 + \frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}} = \\
&-\Bigg(\Bigg(2.77778\Bigg(-1.5625\tan^{-1}(x) - 1.5625\,\pi\left\lfloor\frac{\arg\left(i\left(-x+\frac{1.9625}{\sqrt{2}}\right)\right)}{2\pi}\right\rfloor - \\
&\qquad 0.936\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + \\
&\qquad \exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\log(165.44) \\
&\qquad\quad \sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!} - \\
&\qquad 0.78125\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(-x+\frac{1.9625}{\sqrt{2}}\right)^k}{k} - \\
&\qquad \exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\Bigg(\sqrt{x} \\
&\qquad\qquad \sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,(-0.00604449)^{k_1}\,(2-x)^{k_2}\,x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}}{k_2!\,k_1}\Bigg)\Bigg)\Bigg) \\
&\quad \Bigg/\Bigg(\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x} \\
&\quad \sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\Bigg)\Bigg)
\end{aligned}
$$

for ($i\,x \in \mathbb{R}$ and $i\,x < -1$ and $x \in \mathbb{R}$ and $x < 0$)

---

$$
\begin{aligned}
&2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=\\
&-\Bigg(\Bigg(2.77778\Bigg(-1.5625\tan^{-1}(x)+1.5625\,\pi\left\lfloor\frac{\arg\left(i\left(x-\frac{1.9625}{\sqrt{2}}\right)\right)}{2\pi}\right\rfloor-\\
&\qquad 0.936\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\\
&\qquad \exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\log(165.44)\\
&\qquad\quad \sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-\\
&\qquad 0.78125\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(-x+\frac{1.9625}{\sqrt{2}}\right)^k}{k}-\\
&\qquad \exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\Bigg(\sqrt{x}\\
&\qquad\qquad \sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,(-0.00604449)^{k_1}\,(2-x)^{k_2}\,x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}}{k_2!\,k_1}\Bigg)\Bigg)\Bigg)\\
&\quad \Bigg/\Bigg(\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\\
&\quad \sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\Bigg)\Bigg)
\end{aligned}
$$

for ($i\,x \in \mathbb{R}$ and $i\,x > 1$ and $x \in \mathbb{R}$ and $x < 0$)

## Integral representations

$$
\begin{aligned}
&2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=\\
&\quad 2.6-2.77778\log(166.44)+\frac{8.5178}{\sqrt{2}^{\,2}}\int_0^1\frac{1}{1+\frac{3.85141\,t^2}{\sqrt{2}^{\,2}}}\,dt
\end{aligned}
$$

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$$2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$
$$2.6+\int_{1}^{166.44}\left(-\frac{25}{9\,t}+\frac{365.888}{1-2\,t+t^{2}+7106.6\,\sqrt{2}^{2}}\right)dt$$

---

$$2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$
$$2.6-2.77778\log(166.44)-$$
$$\frac{2.12945\,i}{\pi^{3/2}\,\sqrt{2}^{2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^{2}\left(1+\frac{3.85141}{\sqrt{2}^{2}}\right)^{-s}ds\quad\text{for }0<\gamma<\frac{1}{2}$$

## Continued fraction representations

$$2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$
$$2.6-\frac{459.556}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{165.44\left\lfloor\frac{1+k}{2}\right\rfloor^{2}}{1+k}}+\frac{8.5178}{\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3.85141\,k^{2}}{\sqrt{2}^{2}}}{1+2\,k}\right)\sqrt{2}^{2}}=$$
$$2.6+\frac{4.2589}{1+\frac{1.9257}{3+\frac{7.70281}{5+\frac{17.3313}{7+\frac{30.8113}{9+\dots}}}}}-\frac{459.556}{1+\frac{165.44}{2+\frac{165.44}{3+\frac{661.76}{4+\frac{661.76}{5+\dots}}}}}$$

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$$2.6-\frac{1}{9}\log(2.6\,(9\times2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$2.6-\frac{459.556}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\,\frac{165.44\left\lfloor\frac{1+k}{2}\right\rfloor}{\frac{1}{2}\left(3+(-1)^{k}(-1+k)+k\right)}}+\frac{8.5178}{\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\,\frac{\frac{3.85141\,k^{2}}{\sqrt{2}^{2}}}{1+2k}\right)\sqrt{2}^{2}}=$$

$$2.6+\cfrac{4.2589}{1+\cfrac{1.9257}{3+\cfrac{7.70281}{5+\cfrac{17.3313}{7+\cfrac{30.8113}{9+\dots}}}}}-\cfrac{459.556}{1+\cfrac{165.44}{2+\cfrac{165.44}{3+\cfrac{330.88}{2+\cfrac{330.88}{5+\dots}}}}}$$

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$$2.6-\frac{1}{9}\log(2.6\,(9\times2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$2.6-\frac{459.556}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\,\frac{165.44\left\lfloor\frac{1+k}{2}\right\rfloor^{2}}{1+k}}-\frac{32.8055}{\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\,\frac{\frac{3.85141\left(1+(-1)^{1+k}+k\right)^{2}}{\sqrt{2}^{2}}}{3+2k}\right)\sqrt{2}^{4}}+\frac{8.5178}{\sqrt{2}^{2}}=$$

$$6.8589-\cfrac{8.20137}{3+\cfrac{17.3313}{5+\cfrac{7.70281}{7+\cfrac{48.1426}{9+\cfrac{30.8113}{11+\dots}}}}}-\cfrac{459.556}{1+\cfrac{165.44}{2+\cfrac{165.44}{3+\cfrac{661.76}{4+\cfrac{661.76}{5+\dots}}}}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}}\,a_k/b_k$ is a continued fraction

And:

17.6 -25/9 ln(17.6(9*17.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*17.6+8)/(16sqrt2)))))

**Input**

$$17.6-\frac{25}{9}\log(17.6\,(9\times17.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times17.6+8}{16\sqrt{2}}\right)\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

## Result

−0.354750169164396845468823426569036408394987154482778195166303746
…

(result in radians)

-0.35475016916….

## Alternative representations

$$17.6-\frac{1}{9}\log(17.6\,(9\times17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$
$$17.6-\frac{25\log(3133.44)}{9}+\frac{625\tan^{-1}\left(1,\frac{166.4}{16\sqrt{2}}\right)}{144\sqrt{2}}$$

---

$$17.6-\frac{1}{9}\log(17.6\,(9\times17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$
$$17.6-\frac{25\log_e(3133.44)}{9}+\frac{625\tan^{-1}\left(\frac{166.4}{16\sqrt{2}}\right)}{144\sqrt{2}}$$

---

$$17.6-\frac{1}{9}\log(17.6\,(9\times17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$
$$17.6-\frac{25}{9}\log(a)\log_a(3133.44)+\frac{625\tan^{-1}\left(\frac{166.4}{16\sqrt{2}}\right)}{144\sqrt{2}}$$

## Series representations

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$-\left(\left(2.77778\left(-6.336\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$

$$\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\log(3132.44)$$

$$\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-$$

$$1.5625\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k\,20.8^{1+2\,k}\,F_{1+2\,k}\left(\frac{1}{\sqrt{2}+\sqrt{2}\,\sqrt{1+\frac{86.528}{\sqrt{2}^{\,2}}}}\right)^{1+2\,k}}{1+2\,k}-$$

$$\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\left(\sqrt{x}\right.$$

$$\left.\left.\left.\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,(-0.00031924)^{k_1}\,(2-x)^{k_2}\,x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}}{k_2!\,k_1}\right)\right)\right)$$

$$\left/\left(\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\right.\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\quad \text{for } (x\in\mathbb{R}\text{ and } x<0)$$

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$-\left(\left(2.77778\left(-1.5625\tan^{-1}(x)-1.5625\,\pi\left\lfloor\frac{\arg\left(i\left(-x+\frac{10.4}{\sqrt{2}}\right)\right)}{2\pi}\right\rfloor-\right.\right.\right.$$

$$6.336\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+$$

$$\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\log(3132.44)$$

$$\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-$$

$$0.78125\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(-x+\frac{10.4}{\sqrt{2}}\right)^k}{k}-$$

$$\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(\sqrt{x}\right.$$

$$\left.\left.\left.\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,(-0.00031924)^{k_1}\,(2-x)^{k_2}\,x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}}{k_2!\,k_1}\right)\right)\right)$$

$$\left/\left(\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for ($i\,x\in\mathbb{R}$ and $i\,x<-1$ and $x\in\mathbb{R}$ and $x<0$)

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$-\left(\left(2.77778\left(-1.5625\tan^{-1}(x)+1.5625\,\pi\left\lfloor\frac{\arg\left(i\left(x-\frac{10.4}{\sqrt{2}}\right)\right)}{2\pi}\right\rfloor-\right.\right.\right.$$

$$6.336\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+$$

$$\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\log(3132.44)$$

$$\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-$$

$$0.78125\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(-x+\frac{10.4}{\sqrt{2}}\right)^k}{k}-$$

$$\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(\sqrt{x}\right.$$

$$\left.\left.\left.\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,(-0.00031924)^{k_1}\,(2-x)^{k_2}\,x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}}{k_2!\,k_1}\right)\right)\right)$$

$$\left/\left(\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for ($i\,x \in \mathbb{R}$ and $i\,x > 1$ and $x \in \mathbb{R}$ and $x < 0$)

## Integral representations

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$17.6-2.77778\log(3133.44)+\frac{45.1389}{\sqrt{2}^{\,2}}\int_0^1\frac{1}{1+\frac{108.16\,t^2}{\sqrt{2}^{\,2}}}\,dt$$

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$17.6+\int_1^{3133.44}\left(-\frac{25}{9\,t}+\frac{1307.27}{1-2\,t+t^2+90\,719.1\,\sqrt{2}^{\,2}}\right)dt$$

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$17.6-2.77778\log(3133.44)-$$

$$\frac{11.2847\,i}{\pi^{3/2}\,\sqrt{2}^{\,2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\left(1+\frac{108.16}{\sqrt{2}^{\,2}}\right)^{-s}ds\quad\text{for }0<\gamma<\frac{1}{2}$$

## Continued fraction representations

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$17.6-\frac{8701.22}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{3132.44\left\lfloor\frac{1+k}{2}\right\rfloor^2}{1+k}}+\frac{45.1389}{\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{108.16\,k^2}{\sqrt{2}^{\,2}}}{1+2\,k}\right)\sqrt{2}^{\,2}}=$$

$$17.6+\frac{22.5694}{1+\frac{54.08}{3+\frac{216.32}{5+\frac{486.72}{7+\frac{865.28}{9+\dots}}}}}-\frac{8701.22}{1+\frac{3132.44}{2+\frac{3132.44}{3+\frac{12\,529.8}{4+\frac{12\,529.8}{5+\dots}}}}}$$

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$17.6-\frac{8701.22}{1+\mathop{\mathrm{K}}\limits_{k=1}^{\infty}\frac{3132.44\left\lfloor\frac{1+k}{2}\right\rfloor}{\frac{1}{2}\left(3+(-1)^{k}(-1+k)+k\right)}}+\frac{45.1389}{\left(1+\mathop{\mathrm{K}}\limits_{k=1}^{\infty}\frac{\frac{108.16\,k^{2}}{\sqrt{2}^{2}}}{1+2k}\right)\sqrt{2}^{2}}=$$

$$17.6+\cfrac{22.5694}{1+\cfrac{54.08}{3+\cfrac{216.32}{5+\cfrac{486.72}{7+\cfrac{865.28}{9+\ldots}}}}}-\cfrac{8701.22}{1+\cfrac{3132.44}{2+\cfrac{3132.44}{3+\cfrac{6264.88}{2+\cfrac{6264.88}{5+\ldots}}}}}$$

---

$$17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}=$$

$$17.6-\frac{8701.22}{1+\mathop{\mathrm{K}}\limits_{k=1}^{\infty}\frac{3132.44\left\lfloor\frac{1+k}{2}\right\rfloor^{2}}{1+k}}-\frac{4882.22}{\left(3+\mathop{\mathrm{K}}\limits_{k=1}^{\infty}\frac{\frac{108.16\left(1+(-1)^{1+k}+k\right)^{2}}{\sqrt{2}^{2}}}{3+2k}\right)\sqrt{2}^{4}}+\frac{45.1389}{\sqrt{2}^{2}}=$$

$$40.1694-\cfrac{1220.56}{3+\cfrac{486.72}{5+\cfrac{216.32}{7+\cfrac{1352.}{9+\cfrac{865.28}{11+\ldots}}}}}-\cfrac{8701.22}{1+\cfrac{3132.44}{2+\cfrac{3132.44}{3+\cfrac{12529.8}{4+\cfrac{12529.8}{5+\ldots}}}}}$$

$\mathop{\mathrm{K}}\limits_{k=k_1}^{k_2} a_k / b_k$ is a continued fraction

Inverting the sum of the two above expressions, after some calculations, we obtain:

1/(([2.6 -25/9 ln(2.6(9*2.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*2.6+8)/(16sqrt2)))))]+[17.6 -25/9 ln(17.6(9*17.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*17.6+8)/(16sqrt2)))))]))^72

**Input**

$$1\Big/\left(\left(2.6-\frac{25}{9}\log(2.6\,(9\times 2.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)\right)\right)+\left(17.6-\frac{25}{9}\log(17.6\,(9\times 17.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)\right)\right)\right)^{72}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Result**

$1.24433\ldots\times 10^{-69}$

(result in radians)

$1.24433\ldots*10^{-69}$

**Alternative representations**

$$1\Big/\left(\left(\left(2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)+\left(17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)\right)\right)^{72}=$$

$$1\Big/\left(20.2-\frac{25}{9}\log(a)\log_a(166.44)-\frac{25}{9}\log(a)\log_a(3133.44)+\frac{625\tan^{-1}\left(\frac{31.4}{16\sqrt{2}}\right)}{144\sqrt{2}}+\frac{625\tan^{-1}\left(\frac{166.4}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)^{72}$$

$$1\Bigg/\left(\left(2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)+\left(17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)\right)^{72}=$$

$$1\Bigg/\left(20.2-\frac{25}{9}\log(a)\log_a(166.44)-\frac{25}{9}\log(a)\log_a(3133.44)+\frac{625\tan^{-1}\left(1,\ \frac{31.4}{16\sqrt{2}}\right)}{144\sqrt{2}}+\frac{625\tan^{-1}\left(1,\ \frac{166.4}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)^{72}$$

---

$$1\Bigg/\left(\left(2.6-\frac{1}{9}\log(2.6\,(9\times 2.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)+\left(17.6-\frac{1}{9}\log(17.6\,(9\times 17.6+16)+64)\,25+\frac{625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)\right)^{72}=$$

$$\left(\frac{1}{\left(20.2-\frac{25\log(166.44)}{9}-\frac{25\log(3133.44)}{9}+\frac{625\tan^{-1}\left(1,\frac{31.4}{16\sqrt{2}}\right)}{144\sqrt{2}}+\frac{625\tan^{-1}\left(1,\frac{166.4}{16\sqrt{2}}\right)}{144\sqrt{2}}\right)^{72}}=\frac{1}{\left(20.2-\frac{25\log(166.44)}{9}-\frac{25\log(3133.44)}{9}+\frac{625\tan^{-1}\left(1,\frac{1.9625}{\sqrt{2}}\right)}{144\sqrt{2}}+\frac{625\tan^{-1}\left(1,\frac{10.4}{\sqrt{2}}\right)}{144\sqrt{2}}\right)^{72}}\right)$$

We note that:

(1.1056*10^-52)/(299792458)^2

We consider $\Lambda = 1.1056 * 10^{-52}$ that is the value of the Cosmological Constant and 299792458 , that is the value of the speed of light in *m/s* , the terms of the Einstein equation $E = mc^2$ ( $E = 1.1056 * 10^{-52}$ ($\Lambda$ = Dark Energy) ). Thence, we can to obtain the mass *m*:

**Input interpretation**

$$\frac{1.1056 \times 10^{-52}}{299\,792\,458^2}$$

**Result**

$1.2301459019728805386116738651359598467212878595555184012098\ldots \times 10^{-69}$

$1.2301459019\ldots * 10^{-69}$

From the two previous analyzed expressions:

((((450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2 + (450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2)))^48

**Input**

$$\left(\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times 8.7+9\times 8.7^2\right)^2}+\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times 37.6+9\times 37.6^2\right)^2}\right)^{48}$$

**Result**

$3.0354078186632568216366107605966769350712011813380301580071\ldots \times 10^{-69}$

$3.035407818\ldots * 10^{-69}$

And:

1/(([2.6 -25/9 ln(2.6(9*2.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*2.6+8)/(16sqrt2)))))]+[17.6 -25/9 ln(17.6(9*17.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*17.6+8)/(16sqrt2)))))]))^72

**Input**

$$1\Big/\left(\left(2.6-\frac{25}{9}\log(2.6\,(9\times2.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times2.6+8}{16\sqrt{2}}\right)\right)\right)+\left(17.6-\frac{25}{9}\log(17.6\,(9\times17.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times17.6+8}{16\sqrt{2}}\right)\right)\right)\right)^{72}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Result**

$1.24433\ldots\times10^{-69}$

(result in radians)

$1.24433\ldots*10^{-69}$

we obtain, after some calculations:

(16 sqrt(2/3) log^(3/2)(2))/(e^(5/2) log^2(3)) ((1/(1+(zeta(2))^2)(3.035407*10^-69 + 1.24433× 10^-69)+1.24433× 10^-69))

where

$$\frac{16\sqrt{\frac{2}{3}}\log^{3/2}(2)}{e^{5/2}\log^2(3)}$$

= 0.5127284721865…..

**Input interpretation**

$$\frac{16\sqrt{\frac{2}{3}}\log^{3/2}(2)}{e^{5/2}\log^{2}(3)}\left(\frac{1}{1+\zeta(2)^{2}}\left(3.035407\times10^{-69}+1.24433\times10^{-69}\right)+1.24433\times10^{-69}\right)$$

log(x) is the natural logarithm

ζ(s) is the Riemann zeta function

**Result**

$1.23014\ldots\times10^{-69}$

1.23014…*10$^{-69}$

Or:

(sqrt(((((√(10-2√5) -2))⁄((√5-1))))^(Pi/(2+sqrt3/2)) + 0.011303146014)) ((1/(1+(zeta(2))^2)(3.035407*10^-69 + 1.24433× 10^-69)+1.24433× 10^-69))

where ((((√(10-2√5) -2))⁄((√5-1)))) = κ = 8πG ;   G = 0.011303146014

(sqrt(((((√(10-2√5) -2))⁄((√5-1))))^(Pi/(2+sqrt3/2)) + 0.011303146014)) ((1/(1+(zeta(2))^2)(3.035407*10^-69 + 1.24433× 10^-69)+1.24433× 10^-69))

**Input interpretation**

$$\sqrt{\left(\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)^{\pi/\left(2+\frac{\sqrt{3}}{2}\right)}+0.011303146014}\left(\frac{1}{1+\zeta(2)^{2}}\left(3.035407\times10^{-69}+1.24433\times10^{-69}\right)+1.24433\times10^{-69}\right)$$

ζ(s) is the Riemann zeta function

**Result**

$1.23041\ldots\times10^{-69}$

1.23041…*10$^{-69}$

result that is almost equal to the mass *m* obtained from the Einstein equation inserting the data  *E* = 1.1056 * 10$^{-52}$ (Λ = Dark Energy)  and  c = 299792458

We note that, performing the square root of the above expression, after some calculations, we obtain:

((e - log(8))/log(4))*sqrt((sqrt((((((√(10-2√5) -2))⁄((√5-1))))^(Pi/(2+sqrt3/2)) + 0.011303146014)) ((1/(1+(zeta(2))^2)(3.035407*10^-69 + 1.24433× 10^-69)+1.24433× 10^-69)))

where

$$\frac{e-\log(8)}{\log(4)} \approx 0.4608258568$$

**Input interpretation**

$$\frac{e-\log(8)}{\log(4)} \sqrt{\left(\sqrt{\left(\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)^{\pi/\left(2+\frac{\sqrt{3}}{2}\right)}+0.011303146014}\right.} \left.\left(\frac{1}{1+\zeta(2)^2}\left(3.035407\times 10^{-69}+1.24433\times 10^{-69}\right)+1.24433\times 10^{-69}\right)\right)$$

log(*x*) is the natural logarithm

*ζ*(*s*) is the Riemann zeta function

**Result**

$1.61645\ldots \times 10^{-35}$

$1.61645\ldots{*}10^{-35}$ result almost equal to the value of the Planck length $1.616255{*}10^{-35}$ m

It is possible that can to exist a light particle (possible Dark Matter candidate) having a mass ≈ $10^{-69}$ ?

## Mathematical connections with some sectors of String Theory

## Observations

We note that, from the number 8, we obtain as follows:

$8^2$

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

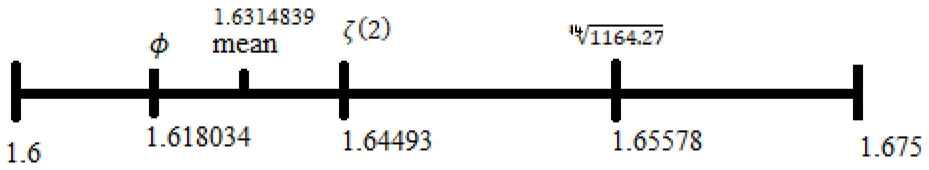
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

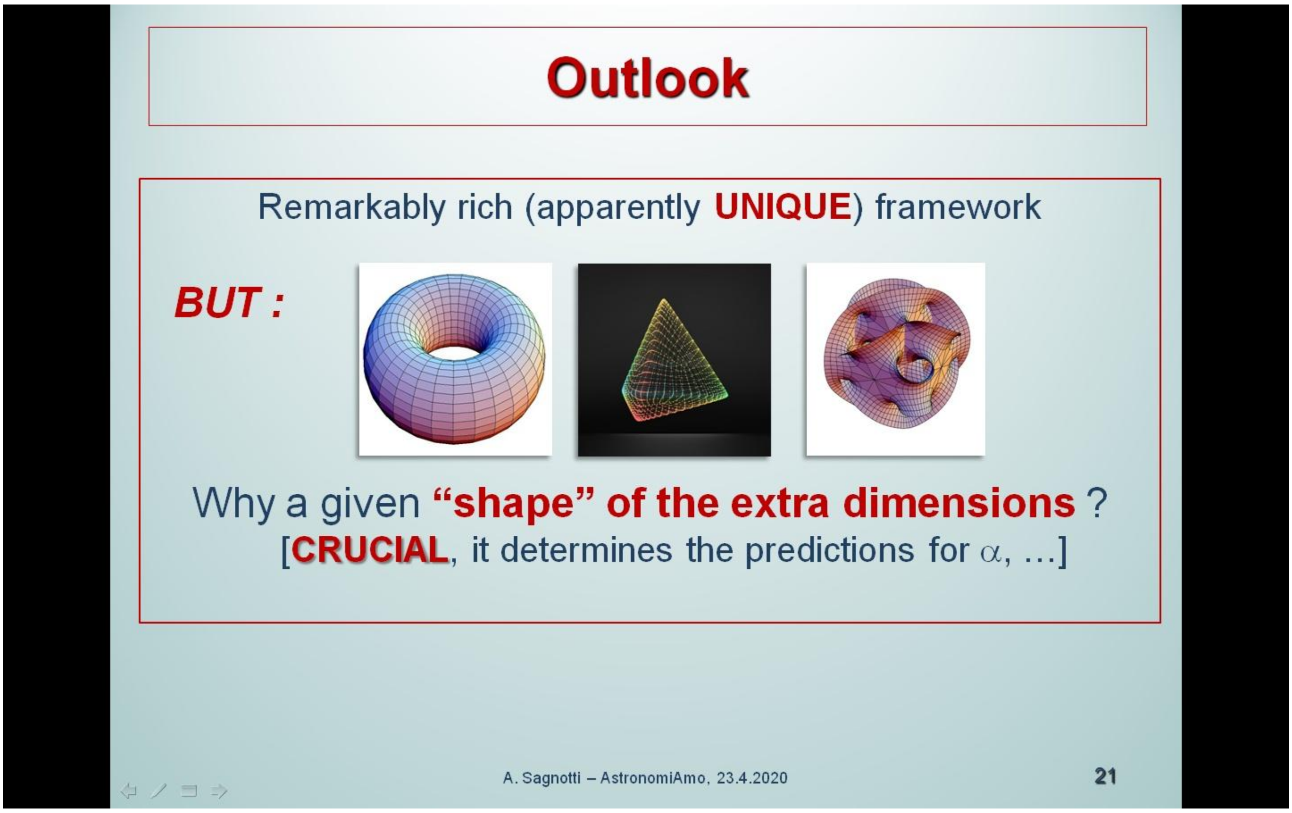
"Golden" Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to ζ(2) that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

We have that:



**Outlook**

Remarkably rich (apparently **UNIQUE**) framework

***BUT :***

Why a given **“shape” of the extra dimensions** ?

[**CRUCIAL**, it determines the predictions for $\alpha$, …]

A. Sagnotti – AstronomiAmo, 23.4.2020

21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: “why a given shape of the extra dimensions? Crucial, it determines the predictions for α”.

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We obtain, in certain cases, the following connections:

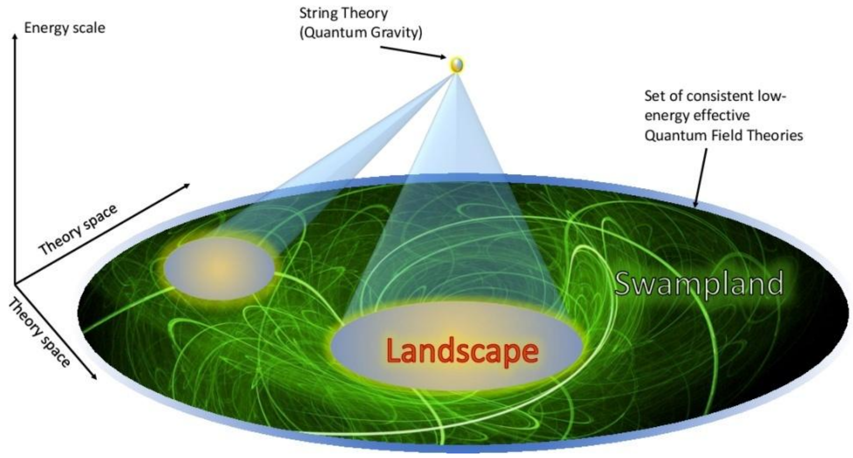
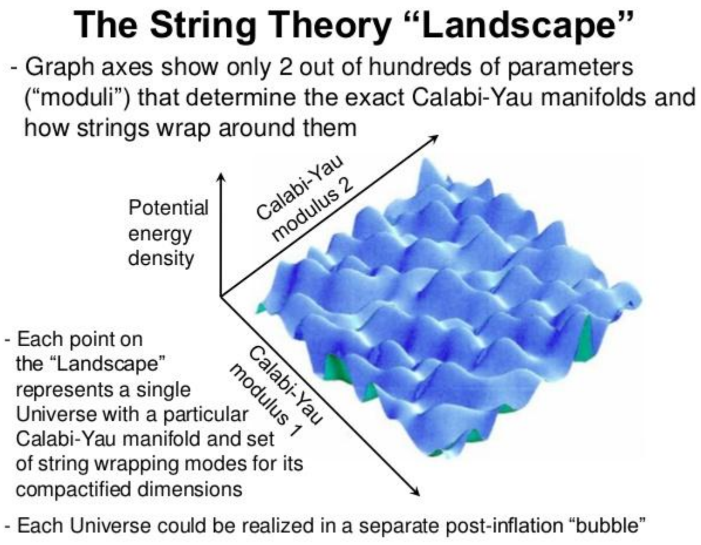


Fig. 1



The String Theory "Landscape"

- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them

- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions

- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

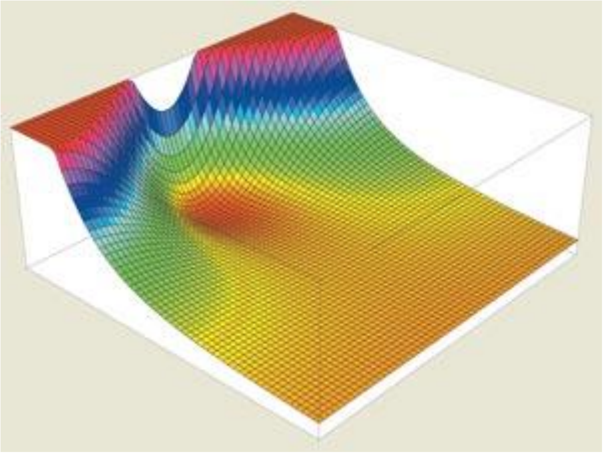


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

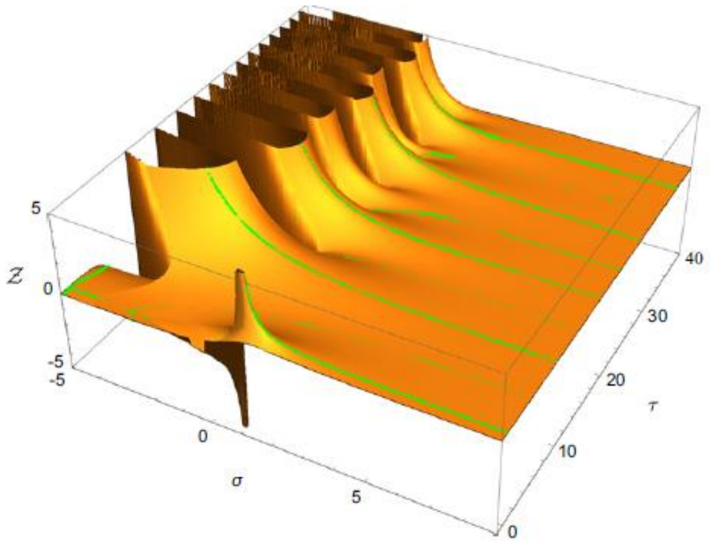


**Figure 2.** Lines in the complex plane where the Riemann zeta function $\zeta$ is real (green) depicted on a relief representing the positive absolute value of $\zeta$ for arguments $s \equiv \sigma + \mathrm{i}\tau$ where the real part of $\zeta$ is positive, and the negative absolute value of $\zeta$ where the real part of $\zeta$ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of $2\pi$ run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \, \frac{\pi}{\ln 2},$$

$$\text{with } k = \ldots, -2, -1, 0, 1, 2, \ldots.$$

we obtain:

2Pi/(ln(2))

**Input:**

$$2 \times \frac{\pi}{\log(2)}$$

**Exact result:**

$$\frac{2\pi}{\log(2)}$$

**Decimal approximation:**

9.0647202836543876192553658914333336203437229354475911683720330958
…

9.06472028365….

**Alternative representations:**

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

## Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

---

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

---

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

## Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

---

$$\frac{2\pi}{\log(2)} = \frac{4 i \pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

**Input:**

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\,\pi\log(2)\right)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{\pi^2}{6}$$

**Decimal approximation:**

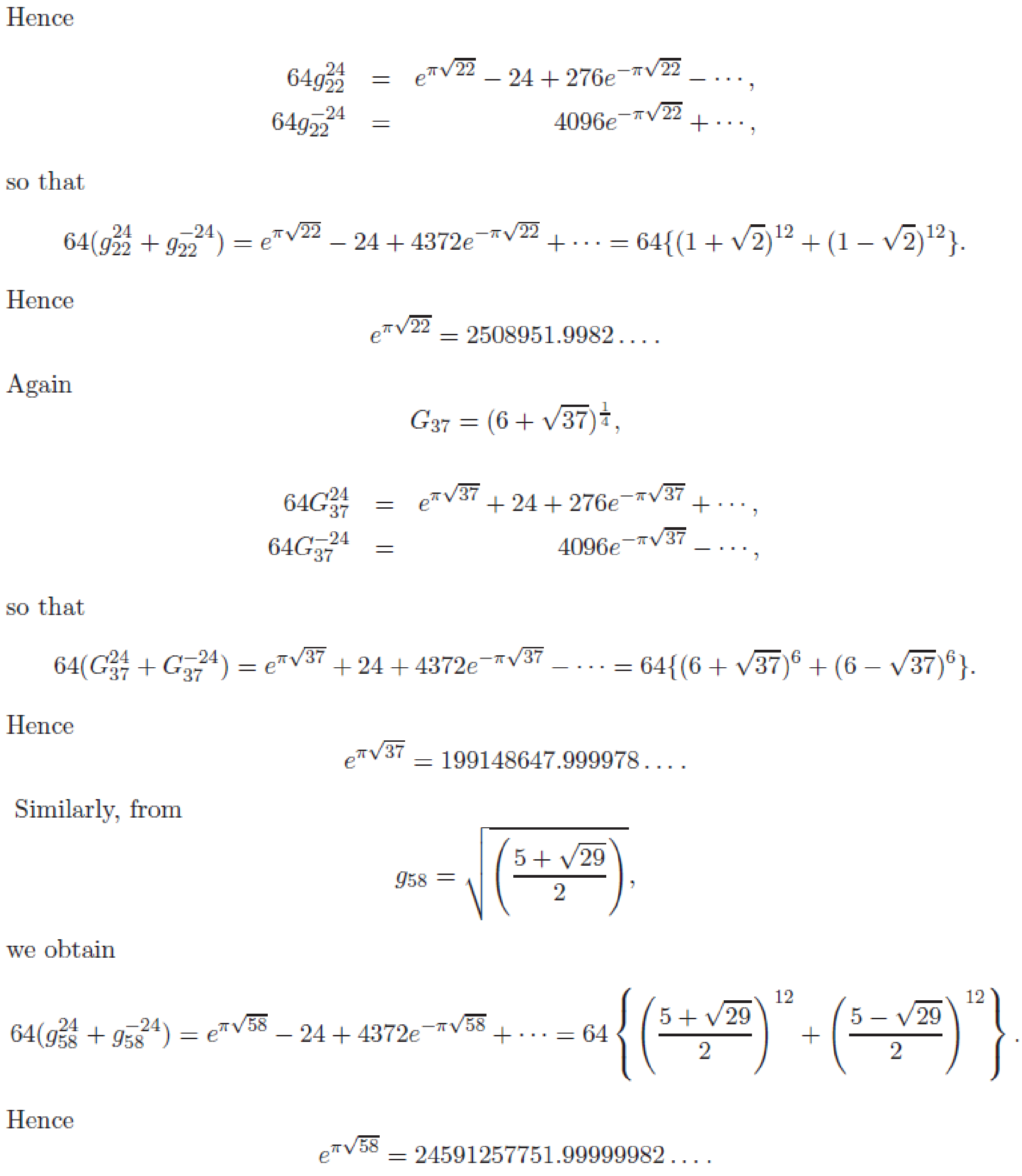
1.6449340668482264364724151666460251892189499012067984377355582293
…

$1.6449340668.... = \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

From:

**Modular equations and approximations to π** - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:



Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

### Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

$\phi$ is the golden ratio

### Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

### Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

…

1729.0526944….

This result is very near to the mass of candidate glueball **f$_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2$ * $3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

### Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

---

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

---

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2\left(\sqrt{5} - 1\right)}$$

## Minimal polynomial

$$\begin{aligned}&256x^8 + 95\,744x^7 - 3\,248\,750\,080x^6 - \\ &\quad 914\,210\,725\,504x^5 + 15\,498\,355\,554\,921\,184x^4 + \\ &\quad 2\,911\,478\,392\,539\,914\,656x^3 - 32\,941\,144\,911\,224\,677\,091\,680x^2 - \\ &\quad 3\,092\,528\,914\,069\,760\,354\,714\,456x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041\end{aligned}$$

## Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

---

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

## Series representations

$$\begin{aligned}&27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \\ &\quad\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + \right. \\ &\qquad 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - \\ &\qquad \left. 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Bigg/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\end{aligned}$$

---

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

## Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

$\phi$ is the golden ratio

## Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

## Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

…

1729.0526944…. as above

## Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

---

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

---

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

## Minimal polynomial

$$256 x^8 + 95\,744 x^7 - 3\,248\,750\,080 x^6 - 914\,210\,725\,504 x^5 + 15\,498\,355\,554\,921\,184 x^4 + 2\,911\,478\,392\,539\,914\,656 x^3 - 32\,941\,144\,911\,224\,677\,091\,680 x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456 x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

## Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

---

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

## Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right) 2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Bigg/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

---

$$
\begin{aligned}
&27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi= \\
&\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+\right. \\
&\quad 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}- \\
&\quad \left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/ \\
&\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right)
\end{aligned}
$$

---

$$
\begin{aligned}
&27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi= \\
&\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}+\right. \\
&\quad 108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}+ \\
&\quad 2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}- \\
&\quad \left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(10-2\sqrt{5}-z_0\right)^{k}z_0^{-k}}{k!}\right)\Big/ \\
&\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}\right)\right)
\end{aligned}
$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ)^1/15

**Input**

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

$\phi$ is the golden ratio

**Exact result**

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

**Decimal approximation**

1.6438185685849862799902301317036810054185756873505184804834183124
…

$1.64381856858\ldots. \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

**Alternate forms**

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

---

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

---

$$\sqrt[15]{\boxed{\begin{array}{l}\text{root of } 256x^8+95\,744x^7-3\,248\,750\,080x^6-914\,210\,725\,504x^5+\\ \quad 15\,498\,355\,554\,921\,184x^4+2\,911\,478\,392\,539\,914\,656x^3-\\ \quad 32\,941\,144\,911\,224\,677\,091\,680x^2-3\,092\,528\,914\,069\,760\,354\,714\,456x+\\ \quad 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x=1729.05\end{array}}}$$

## Minimal polynomial

$$\begin{array}{l}256x^{120}+95\,744x^{105}-3\,248\,750\,080x^{90}-\\ \quad 914\,210\,725\,504x^{75}+15\,498\,355\,554\,921\,184x^{60}+\\ \quad 2\,911\,478\,392\,539\,914\,656x^{45}-32\,941\,144\,911\,224\,677\,091\,680x^{30}-\\ \quad 3\,092\,528\,914\,069\,760\,354\,714\,456x^{15}+26\,320\,050\,609\,744\,039\,027\,169\,013\,041\end{array}$$

## Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right)+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

---

$$\sqrt[15]{-\frac{187}{4}+\frac{29\sqrt{5}}{4}+54\sqrt{1093}-\frac{27}{8}\sqrt{10-2\sqrt{5}}-\frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

## All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.64382 \text{ (real, principal root)}$$

---

$$e^{(2i\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.50170+0.6686\,i$$

---

$$e^{(4 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 1.0999 + 1.2216\, i$$

---

$$e^{(2 i \pi)/5} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 0.5080 + 1.5634\, i$$

---

$$e^{(8 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx -0.17183 + 1.63481\, i$$

---

## Series representations

$$\sqrt[15]{27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi} =$$

$$\frac{1}{\sqrt[15]{2}} \left(\left(\left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(9 - 2\sqrt{5}\right)^{-k}\right) \middle/ \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)\right)\right) \wedge (1/15)\right)$$

---

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$

$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$

$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$

$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)^\wedge(1/15)\right)$$

---

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$

$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$

$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$

$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$

$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)^\wedge(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

## Integral representation

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From:

**An Update on Brane Supersymmetry Breaking**
*J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\,\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{-\,2\,C} \;=\; \frac{h^2 \left(p \,+\, 1 \,-\, \frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\, e^{-\,2\,A} \,+\, \frac{h^2}{16(p\,+\,1)} \left(7 \,-\, p \,+\, \frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning $p$, $C$, $\beta_E$ and $\phi$ correspond to the exponents of $e$ (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to $64^2$, while -6C+$\phi$ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

**Input:**

$$\exp\left(-\pi\sqrt{18}\right)$$

**Exact result:**

$$e^{-3\sqrt{2}\,\pi}$$

**Decimal approximation:**

$$1.6272016226072509292942156739117979541838581136954016\ldots \times 10^{-6}$$

$1.6272016\ldots * 10^{-6}$

**Property:**

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

**Series representations:**

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

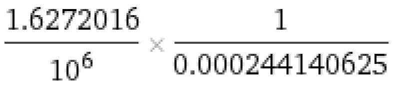
Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

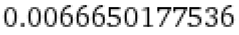
(1.6272016* 10^-6) *1/ (0.000244140625)

**Input interpretation:**



$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

**Result:**



0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625}\, e^{-6C+\phi} = \frac{1}{0.000244140625}\, e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

**Input interpretation:**

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625}$$

**Result:**

$$0.00666501785\ldots$$

0.00666501785…

**Series representations:**

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

**Input interpretation:**

$\log(0.00666501784619)$

**Result:**

$-5.010882647757\ldots$

-5.010882647757…

## Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

## Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

## Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$

Note that the values of $n_s$ (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.9874428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}}-1}-\varphi+1} = 1 - \cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

The mean between the two results of the above Rogers-Ramanujan continued fractions is 0.97798855285, value very near to the ψ Regge slope 0.979:

| | | | | |
|---|---|---|---|---|
| $\Psi$ | 3 | $m_c = 1500$ | 0.979 | $-0.09$ |

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

**Input interpretation:**

$$\sqrt[512]{\frac{1}{139.57}}$$

**Result:**

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value **0.989117352243 = $\boldsymbol{\phi}$** and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

**AdS Vacua from Dilaton Tadpoles and Form Fluxes -** *J. Mourad and A. Sagnotti* - arXiv:1612.08566v2 [hep-th] 22 Feb 2017 - March 27, 2018

We have:

$$\begin{aligned} e^{2C} &= \frac{2\,\xi\, e^{\frac{\phi}{2}}}{1 \,\pm\, \sqrt{1 \,-\, \frac{\xi T}{3}\, e^{2\phi}}} \\ \frac{h^2}{32} &= \frac{\xi^7\, e^{4\phi}}{\left(1 \,\pm\, \sqrt{1 \,-\, \frac{\xi T}{3}\, e^{2\phi}}\right)^7} \left[\frac{42}{\xi}\left(1 \,\pm\, \sqrt{1 \,-\, \frac{\xi\, T}{3}\, e^{2\phi}}\right) \,+\, 5\, T\, e^{2\phi}\right] . \end{aligned} \tag{2.7}$$

For

$T = \frac{16}{\pi^2}$

ξ = 1

we obtain:

(2*e^(0.989117352243/2)) / (1+sqrt(((1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))

**Input interpretation:**

$$\frac{2\, e^{0.989117352243/2}}{1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\, e^{2\times 0.989117352243}}}$$

**Result:**

$0.83941881822\ldots -$
$1.4311851867\ldots\, i$

**Polar coordinates:**

$r = 1.65919106525$ (radius), $\theta = -59.607521917°$ (angle)

1.65919106525….. result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

**Series representations:**

$$\frac{2\, e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\, e^{2\times 0.98911735224300000}}{3\, \pi^2}}} = \frac{2\, e^{0.494558676121500000}}{1+\sqrt{-\frac{16\, e^{1.978234704486000}}{3\, \pi^2}}\ \sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{2\, e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\, e^{2\times 0.98911735224300000}}{3\, \pi^2}}} = \frac{2\, e^{0.494558676121500000}}{1+\sqrt{-\frac{16\, e^{1.978234704486000}}{3\, \pi^2}}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{2\, e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\, e^{2\times 0.98911735224300000}}{3\, \pi^2}}} = \frac{2\, e^{0.494558676121500000}}{1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\, e^{1.978234704486000}}{3\, \pi^2}-z_0\right)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From

$$\frac{h^2}{32} = \frac{\xi^7 e^{4\phi}}{\left(1 \pm \sqrt{1-\frac{\xi T}{3}e^{2\phi}}\right)^7}\left[\frac{42}{\xi}\left(1 \pm \sqrt{1-\frac{\xi T}{3}e^{2\phi}}\right) + 5\, T e^{2\phi}\right]$$

we obtain:

e^(4*0.989117352243) / (((1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))^7 [42(1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))+5*16/(Pi)^2*e^(2*0.989117352243)]

**Input interpretation:**

$$\frac{e^{4\times 0.989117352243}}{\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)+5\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}\right)$$

**Result:**

50.84107889... –
20.34506335... $i$

**Polar coordinates:**

$r = 54.76072411$ (radius), $\theta = -21.80979492°$ (angle)

54.76072411…..

**Series representations:**

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Bigg/\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7 =$$

$$\left(2\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Bigg/\left(\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7=$$

$$\left(2\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/$$

$$\left(\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7=$$

$$\left(2\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Big/$$

$$\left(\pi^2\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

e^(4*0.989117352243) / (((1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243))))))^7 [42(1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))+5*16/(Pi)^2*e^(2*0.989117352243)]*1/34

## Input interpretation:

$$\frac{e^{4\times 0.989117352243}}{\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)+5\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}\right)\times\frac{1}{34}$$

## Result:

$1.495325850\ldots -$
$0.5983842161\ldots\, i$

## Polar coordinates:

$r = 1.610609533$ (radius), $\theta = -21.80979492°$ (angle)

1.610609533…. result that is a good approximation to the value of the golden ratio 1.618033988749...

## Series representations:

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Bigg/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7\right)=$$

$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\Bigg/\left(17\,\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Bigg/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7\right)=$$

$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Bigg/$$

$$\left(17\,\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Bigg/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7\right)=$$

$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)\Bigg/$$

$$\left(17\,\pi^2\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Now, we have:

$$e^{2C} = \frac{2\,\xi\,e^{-\frac{\phi}{2}}}{1 + \sqrt{1 + \frac{\xi\Lambda}{3}\,e^{2\phi}}}\,, \tag{2.9}$$

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1 + \sqrt{1 + \frac{\Lambda}{3}\,e^{2\phi}}\right]^7}\left[42\left(1 + \sqrt{1 + \frac{\Lambda}{3}\,e^{2\phi}}\right) - 13\,\Lambda\,e^{2\phi}\right]. \tag{2.10}$$

For:

$\xi = 1$

$\Lambda \simeq \frac{4\pi^2}{25}$

$\phi = 0.989117352243$

From

$$e^{2C} = \frac{2\,\xi\,e^{-\frac{\phi}{2}}}{1 + \sqrt{1 + \frac{\xi\Lambda}{3}\,e^{2\phi}}}\,,$$

we obtain:

((2*e^(-0.989117352243/2))) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))))))

**Input interpretation:**

$$\frac{2\,e^{-0.989117352243/2}}{1 + \sqrt{1 + \frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times 0.989117352243}}}$$

**Result:**

0.382082347529...

0.382082347529….

**Series representations:**

$$\frac{2 e^{-0.98911735224300000/2}}{1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.98911735224300000}}{3\times 25}}} = 2\Bigg/\left(e^{0.49455867612150000}\left(1+\sqrt{\frac{4 e^{1.978234704486000}\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$\frac{2 e^{-0.98911735224300000/2}}{1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.98911735224300000}}{3\times 25}}} = 2\Bigg/\left(e^{0.49455867612150000}\left(1+\sqrt{\frac{4 e^{1.978234704486000}\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{2 e^{-0.98911735224300000/2}}{1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.98911735224300000}}{3\times 25}}} = \frac{2}{e^{0.49455867612150000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4 e^{1.978234704486000}\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)}$$

$$\text{for }(\text{not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$$

From which:

1+1/(((4((2*e^(-0.989117352243/2))) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))))))))

**Input interpretation:**

$$1+\frac{1}{4\times\frac{2 e^{-0.989117352243/2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}(4\pi^2)\right)e^{2\times 0.989117352243}}}}$$

**Result:**

1.65430921270...

1.6543092….. We note that, the result 1.6543092... is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Indeed:

$$G_{505} = P^{-1/4}Q^{1/6} = (\sqrt{5}+2)^{1/2}\left(\frac{\sqrt{5}+1}{2}\right)^{1/4}(\sqrt{101}+10)^{1/4}$$
$$\times\left((130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}\right)^{1/6}.$$

Thus, it remains to show that

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}} = \left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

which is straightforward. □

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}65578\ldots$$

**Series representations:**

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}} =$$

$$1+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8}\,e^{0.4945586761215000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}$$
$$\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.9891173522430000/2}\right)}{1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times0.9891173522430000}}{3\times25}}}} =$$

$$1+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8}\,e^{0.4945586761215000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}$$

$$\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.9891173522430000/2}\right)}{1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times0.9891173522430000}}{3\times25}}}} = 1+\frac{e^{0.4945586761215000}}{8}+$$

$$\frac{1}{8}\,e^{0.4945586761215000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

And from

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1+\sqrt{1+\frac{\Lambda}{3}e^{2\phi}}\right]^7}\left[42\left(1+\sqrt{1+\frac{\Lambda}{3}e^{2\phi}}\right)-13\,\Lambda\,e^{2\phi}\right].$$

we obtain:

e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))-13*(4Pi^2)/25*e^(2*0.989117352243)]

## Input interpretation:

$$\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}}\right)^{7}}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}}\right)-13\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}\right)$$

## Result:

−0.034547055658...

-0.034547055658…

## Series representations:

$$\left(\left(42\left(1+\sqrt{1+\frac{\left(4\pi^{2}\right)e^{2\times 0.9891173522430000}}{3\times 25}}\right)-\frac{1}{25}\left(4\pi^{2}\right)13\,e^{2\times 0.9891173522430000}\right)e^{-4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{\left(4\pi^{2}\right)e^{2\times 0.9891173522430000}}{3\times 25}}\right)^{7}=$$

$$-\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^{2}-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^{2}}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000}\,\pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^{2}}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000}\,\pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{7}\right)\right)$$

$$\left(\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)e^{-4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 = -\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)$$

$$\left(\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)e^{-4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 = -\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)$$

$\text{for }(\text{not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$

From which:

47 *1/(((-1/((((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))))))

**Input interpretation:**

$$47\left(-\left(1\Big/1\Big/\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times 0.989117352243}}\right)-13\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)$$

**Result:**

1.6237116159...

1.6237116159…. result that is an approximation to the value of the golden ratio 1.618033988749...

**Series representations:**

$$-\left(47\Big/1\Big/\left(e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times 0.9891173522430000}}{3\times 25}}\right)-\frac{1}{25}\left(4\,\pi^2\right)13\,e^{2\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)\right)=$$

$$\left(1974\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$-(47/1/(e^{-4\times 0.9891173522430000}(42(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}))))/ ((1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}})^7))=$$

$$(1974(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}))/(25\,e^{5.934704113458000}(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!})^7)$$

$$-(47/1/(e^{-4\times 0.9891173522430000}(42(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}))))/ ((1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}})^7))=$$

$$(1974(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}))/(25\,e^{5.934704113458000}(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!})^7)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

And again:

32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))- 13*(4Pi^2)/25*e^(2*0.989117352243))]))))

**Input interpretation:**

$$32\left(\frac{e^{-4\times0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times0.989117352243}\right)\right)\right)$$

**Result:**

−1.1055057810...

-1.1055057810….

We note that the result -1.1055057810…. is very near to the value of Cosmological Constant, less $10^{-52}$ , thence 1.1056, with minus sign

**Series representations:**

$$\left(32\, e^{-4\times 0.9891173522430000} \left(42 \left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}} - \frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right) \Big/ \left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 =$$

$$-\left(\left(\left(1344 \left(-25\, e^{1.978234704486000} + 52\, e^{3.956469408972000}\, \pi^2 - 25\, e^{1.978234704486000} \sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \binom{\frac{1}{2}}{k}\right)\right)\right) \Big/ \left(25\, e^{5.934704113458000} \left(1+\sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \binom{\frac{1}{2}}{k}\right)^7\right)\right)$$

$$\left(32\, e^{-4\times 0.9891173522430000} \left(42 \left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}} - \frac{1}{25}\,(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right) \Big/ \left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 = -\left(\left(1344 \left(-25\, e^{1.978234704486000} + 52\, e^{3.956469408972000}\, \pi^2 - 25\, e^{1.978234704486000} \sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)\right) \Big/ \left(25\, e^{5.934704113458000} \left(1+\sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)$$

$$\left(32\, e^{-4\times 0.9891173522430000} \left(42 \left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}} - \frac{1}{25}\,(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right) \Big/ \left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 = -\left(\left(1344 \left(-25\, e^{1.978234704486000} + 52\, e^{3.956469408972000}\, \pi^2 - 25\, e^{1.978234704486000} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1+\frac{4\, e^{1.978234704486000}\, \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!}\right)\right) \Big/ \left(25\, e^{5.934704113458000} \left(1+\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1+\frac{4\, e^{1.978234704486000}\, \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

And:

-[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))]^5

**Input interpretation:**

$$-\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)^5$$

**Result:**

1.651220569...

1.651220569…. result very near to the 14th root of the following Ramanujan’s class invariant $Q=\left(G_{505}/G_{101/5}\right)^3=1164.2696$ i.e. 1.65578...

## Series representations:

$$-\left(\left(\left(32\, e^{-4 \times 0.9891173522430000} \left(42 \left(1+\sqrt{1+\frac{\left(4 \pi^2\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\; - \frac{1}{25} \left(4 \pi^2\right) 13\, e^{2 \times 0.9891173522430000}\right)\right)\right)\right) \Big/ \left(1+\sqrt{1+\frac{\left(4 \pi^2\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^7\right)^5 =$$

$$\left(4\,385\,270\,057\,140\,224 \left(-25+52\, e^{1.978234704486000}\, \pi^2-25 \sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \binom{\frac{1}{2}}{k}\right)^5\right) \Big/ \left(9\,765\,625\, e^{19.78234704486000} \left(1+\sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \binom{\frac{1}{2}}{k}\right)^{35}\right)$$

$$-\left(\left(\left(32\,e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\,(4\,\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)^5 = \left(4\,385\,270\,057\,140\,224\left(-25+52\,e^{1.978234704486000}\,\pi^2-25\,\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^5\right)\Big/\left(9\,765\,625\,e^{19.78234704486000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{35}\right)$$

$$-\left(\left(\left(32\,e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\,(4\,\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)^5 = \left(4\,385\,270\,057\,140\,224\left(-25+52\,e^{1.978234704486000}\,\pi^2-25\,\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^5\right)\Big/\left(9\,765\,625\,e^{19.78234704486000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^{35}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We obtain also:

-[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))]^1/2

**Input interpretation:**

$$-\sqrt{\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)}$$

**Result:**

$-0$
$1.0514303501\ldots\, i$

**Polar coordinates:**

$r = 1.05143035007$ (radius), $\theta = -90°$ (angle)

1.05143035007

## Series representations:

$$-\sqrt{\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)} = -\frac{8}{5}\sqrt{21}\sqrt{\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)}$$

$$-\sqrt{\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\,\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\,\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\,\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)}=-\frac{8}{5}\sqrt{21}\,\sqrt{\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)}$$

$$-\sqrt{\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\,\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\,\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\,\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)}=-\frac{8}{5}\sqrt{21}\,\sqrt{\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

1 / -[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))- 13*(4Pi^2)/25*e^(2*0.989117352243))])))))]^1/2

**Input interpretation:**

$$-\left(1\Big/\sqrt{\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)}\right)$$

**Result:**

$0.95108534763\ldots\, i$

**Polar coordinates:**

$r = 0.95108534763$ (radius), $\theta = 90°$ (angle)

0.95108534763

We know that the primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

Thence 0.95108534763 is a result very near to the spectral index $n_s$ , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\dots}}}}\approx 0.9568666373$$

## Series representations:

$$-\left(1\Bigg/\left(\sqrt{\left(\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Bigg/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)\right)}\right)\right)=$$

$$-\left(5\Bigg/\left(8\sqrt{21}\,\sqrt{\left(\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\Bigg/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)\right)}\right)\right)$$

$$
\begin{aligned}
&-\Bigg(1\Bigg/\Bigg(\sqrt{\Bigg(\Bigg(\Bigg(32\,e^{-4\times 0.9891173522430000}\Bigg(42\Bigg(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\Bigg)-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\Bigg)^7\Bigg)}\Bigg)\Bigg)=\\
&-\Bigg(5\Bigg/\Bigg(8\sqrt{21}\sqrt{\Bigg(\Bigg(\Bigg(25-52\,e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\Bigg)\Bigg/\Bigg(e^{3.956469408972000}\Bigg(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\Bigg)^7\Bigg)\Bigg)\Bigg)}\Bigg)\Bigg)
\end{aligned}
$$

$$-\left(1/\left(\sqrt{\left(\left(32\,e^{-4\times0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times0.9891173522430000}}{3\times25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times0.9891173522430000}\right)\right)\right)/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)}\right)\right)=$$

$$-\left(5/\left(8\sqrt{21}\sqrt{\left(\left(25-52\,e^{1.978234704486000}\,\pi^2+25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)/\left(e^{3.956469408972000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)}\right)\right)$$

for $(\text{not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$

From the previous expression

$$\frac{e^{-4\times0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}(4\pi^2)\right)e^{2\times0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}(4\pi^2)\right)e^{2\times0.989117352243}}-13\left(\frac{1}{25}(4\pi^2)\right)e^{2\times0.989117352243}\right)\right)$$

$= -0.034547055658\ldots$

we have also:

1+1/(((4((2*e^(-0.989117352243/2))) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))))))))) + (-0.034547055658)

**Input interpretation:**

$$1+\frac{1}{4\times\frac{2\,e^{-0.989117352243/2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times 0.989117352243}}}}-0.034547055658$$

**Result:**

1.61976215705...

1.61976215705….. result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Series representations:**

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times 0.98911735224300000}}{3\times 25}}}}-0.0345470556580000=$$

$$0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8}\,e^{0.4945586761215000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times 0.98911735224300000}}{3\times 25}}}}-0.0345470556580000=$$

$$0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8}\,e^{0.4945586761215000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.989117352243000/2}\right)}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.989117352243000}}{3\times 25}}}}-0.0345470556580000=$$

$$0.965452944342000+\frac{e^{0.4945586761215000}}{8}+$$

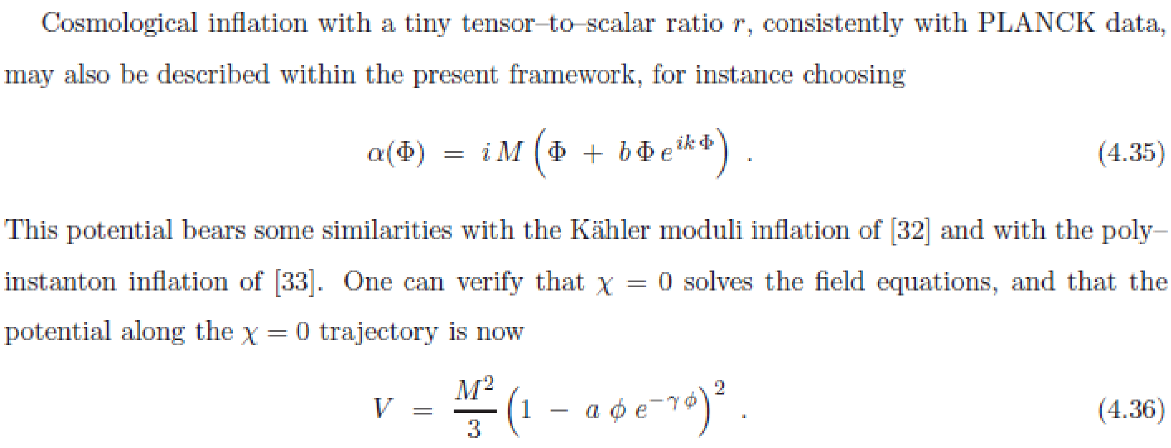
$$\frac{1}{8}\,e^{0.4945586761215000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From

**Properties of Nilpotent Supergravity**
*E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:



Cosmological inflation with a tiny tensor–to–scalar ratio $r$, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi)\;=\;i\,M\left(\Phi\;+\;b\,\Phi\,e^{ik\,\Phi}\right)\;. \qquad (4.35)$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi=0$ solves the field equations, and that the potential along the $\chi=0$ trajectory is now

$$V\;=\;\frac{M^2}{3}\left(1\;-\;a\,\phi\,e^{-\gamma\,\phi}\right)^2\;. \qquad (4.36)$$

We analyzing the following equation:

$$V\;=\;\frac{M^2}{3}\left(1\;-\;a\,\phi\,e^{-\gamma\,\phi}\right)^2\;.$$

$$\phi\;=\;\varphi\;-\;\frac{\sqrt{6}}{k}\;,$$

$$a = \frac{b\gamma}{e} < 0 , \qquad \gamma = \frac{k}{\sqrt{6}} < 0 .$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

**Input interpretation:**

$$V = \frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

**Result:**

$$V = \frac{1}{3}(0.0814845\,b+1)^2 M^2$$

## Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

## Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

## Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

## Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

## Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

## Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

## Implicit derivatives

$$\frac{\partial b(M, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

---

$$\frac{\partial b(M, V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

---

$$\frac{\partial M(b, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

---

$$\frac{\partial M(b, V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

---

$$\frac{\partial V(b, M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

---

$$\frac{\partial V(b, M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

## Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

**Global minima:**

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\,2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

$$\text{for } b=-\frac{226\,802\,245}{18\,480\,874}$$

---

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\,2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

$$\text{for } M=0$$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

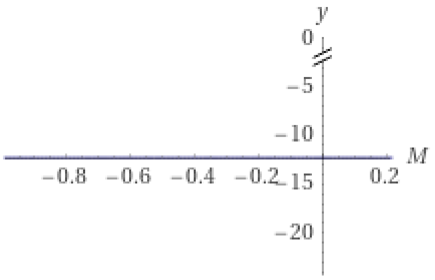
**Input interpretation:**

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

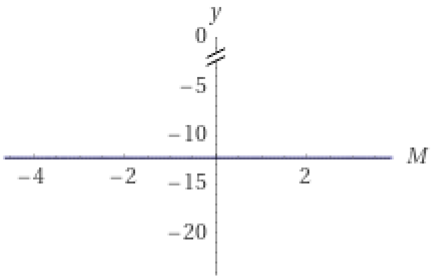
**Result:**

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

## Plots:



(M from −1 to 0.2)



(M from −4.6 to 3.9)

## Alternate form assuming M is real:

$-12.2723$

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

## Alternate forms:

$$-\frac{12.2723\left(M^2 - 1.21228\times 10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times 10^{-7}\sqrt{M^4} - 12.2723\,M^2}{M^2}$$

## Expanded form:

$$\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

## Property as a function:

### Parity

even

## Series expansion at M = 0:

$$\left(\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

## Series expansion at M = ∞:

$$-12.2723$$

## Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times 10^{-15}}{M}$$

## Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$

$$\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

**Global maximum:**

$$\max\left\{\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}\ -0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

**Global minimum:**

$$\min\left\{\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}\ -0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

**Limit:**

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} = -12.2723$$

**Definite integral after subtraction of diverging parts:**

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} - -12.2723\right)dM = 0$$

From  b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2 M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2 ) + 1)^2 M^2

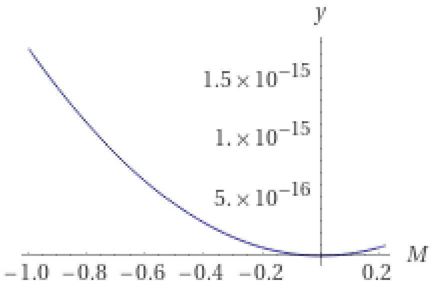
**Input interpretation:**

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

**Result:**

0

**Plots:** **(possible mathematical connection with an open string)**

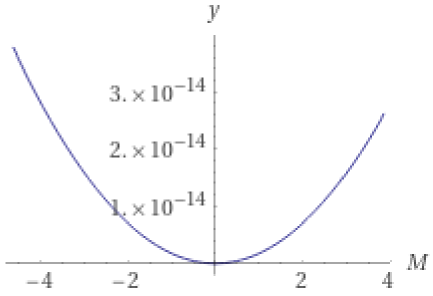


(M from −1 to 0.2)

M = -0.5;  M = 0.2

---

**(possible mathematical connection with an open string)**



(M from −4.6 to 3.9)

M = 2 ; M = 3

## Root:

$M = 0$

## Property as a function:

### Parity

even

## Series expansion at M = 0:

$O(M^{62194})$

(Taylor series)

## Series expansion at M = ∞:

$1.75541 \times 10^{-15} M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$

(Taylor series)

**Definite integral after subtraction of diverging parts:**

$$\int_0^\infty \left(\frac{1}{3} M^2 \left(1 + \frac{18.4084\left(-0.054323\, M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2 - 1.75541\times 10^{-15}\, M^2\right) dM = 0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times \frac{225.913\left(-0.054323\, M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2 ) + 1)^2 * (-0.5^2)

**Input interpretation:**

$$\frac{1}{3}\left(0.0814845\times \frac{225.913\left(-0.054323\,(-0.5)^2 + 6.58545\times 10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2} + 1\right)^2 \left(-0.5^2\right)$$

**Result:**

$$-4.388513449474645453489707833780880208333333333333333333\ldots \times 10^{-16}$$

$-4.38851344947*10^{-16}$

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2 ) + 1)^2 0.2^2

**Input interpretation:**

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

**Result:**

$$7.0216215191594327255835325340494083333333333333333333333333\ldots\times 10^{-17}$$

$7.021621519159*10^{-17}$

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2 ) + 1)^2 3^2

**Input interpretation:**

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 3^2+6.58545\times 10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times 3^2$$

**Result:**

$1.579864841810872363256294820161116875\times 10^{-14}$

$1.57986484181{*}10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2 ) + 1)^2 2^2

**Input interpretation:**

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

**Result:**

$$7.0216215191594327255835325340494083333333333333333333333333\ldots \times 10^{-15}$$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

**Input interpretation:**

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)}$$

**Result:**

$$5.9776991059\ldots \times 10^{-8}$$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow $5.975498 \times 10^{-8}$ that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)])))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

**Input interpretation:**

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right) \wedge (1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\; e \log^{3/2}(3)}\right)$$

log($x$) is the natural logarithm

**Result:**

1.6181818182...

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * $10^{-35}$ Lorentz-Heaviside value

## Planck’s Electric field strength

$$\mathbf{E}_\mathrm{P} = \frac{F_\mathrm{P}}{q_\mathrm{P}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * $10^{61}$ V*m  Lorentz-Heaviside value

## Planck’s Electric flux

$$\phi_\mathrm{P}^E = \mathbf{E}_\mathrm{P} l_\mathrm{P}^2 = \phi_\mathrm{P} l_\mathrm{P} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*$10^{-8}$ V*m  Lorentz-Heaviside value

## Planck’s Electric potential

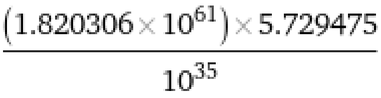
$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*$10^{27}$ V  Lorentz-Heaviside value

## Relationship between Planck’s Electric Flux and  Planck’s Electric Potential

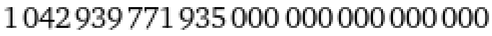
**$E_P$ * $l_P$** = $(1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

**Input interpretation:**



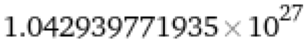
$$\frac{(1.820306\times 10^{61})\times 5.729475}{10^{35}}$$

**Result:**



1 042 939 771 935 000 000 000 000 000

**Scientific notation:**



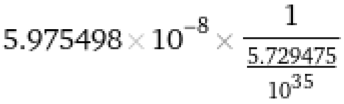
$1.042939771935\times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

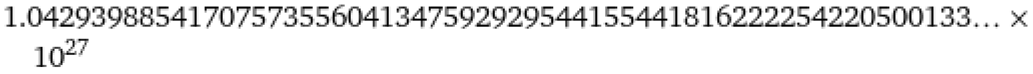
**$E_P$ * $l_P^2$ / $l_P$** = $(5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

**Input interpretation:**



$$5.975498\times 10^{-8}\times \frac{1}{\frac{5.729475}{10^{35}}}$$

**Result:**



$1.04293988541707573556041347592929544155441816222254220500133\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

## Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability

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